STUDY OF TECHNIQUES FOR DISPLAY OF MULTILEVEL PICTURES ON BILEVEL DISPLAYS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

by
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to the
DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

SEPTEMBER, 1982

CERTIFICATE

Certified that the thesis entitled 'Study of Techniques for Display of Multilevel Pictures on Bilevel Displays' has been carried out under my supervision by Capt.S.K. Mohla and has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENTS

I am deeply indebted to Dr. S.K. Mullick for his guidance and help during the period of this work. I am very grateful to him for the understanding, patience and encouragement shown, especially, during the period followi my mother's demise.

I wish to express my sincere thanks to Capt. A.V. Subramanian and Mr. P.G. Poonacha for putting themselves t a lot of trouble over helping me with this work in various ways. Mr. C.M. Abraham deserves thanks for prompt and accurate typing.

Finally, I would like to thank my wife, Sunita for proof reading and diagram making.

S.K. Mohla

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ABSTRACT

This work is a study of processing techniques for presenting continuous tone or computer generated multi gray level pictures on bilevel displays. These displays are bilevel in nature and have individual display cells, all of the same size, arranged in a rectangular array. Six processing techniques have been discussed. All the techniques achieve the subjective effect of continuous tone by properly controlling only the spatial density of bilevel display states.

A nonlinear model of the human visual system has also been considered to show analytically the eye's perception of spatially arranged binary patterns as different gray levels. Two sets of such binary patterns have then been used to display a computer generated picture.

In the remaining processing techniques the input image is divided into picture elements (pixels) and intensity of each pixel is then compared with a threshold value. The corresponding output pixel is turned to bright state if the intensity is greater than the threshold. The processing techniques differ in the manner the threshold value is calculated. Images processed by all the techniques are exhibited, evaluated and compared.

CHAPTER I

INTRODUCTION

This work is a study of techniques/algorithms that can be used to transform a multi-level continuous tone/computer generated still, monochrome image for reproduction on binary displays/printing systems. In other words, the algorithms transform a multi-gray level picture into a bilevel picture which retains the same gray tone effects because of the transformation of the multi-level picture into spatially encoded representations compatible with binary displays. A continuous tone image is one which contains an apparent continuum of gray levels. Some scenes when viewed by the human eye may require upto or more than 256 discrete gray levels to give the appearance of a continuum of gray levels from one shade to another. Examples of continuous tone images are television (CRT) images, photographic images and real world scenes viewed by a camera.

The bilevel/binary output images produced by the algorithms discussed in the subsequent chapters differ from other bilevel images like halftone images, and line copy images. In the bilevel pictures produced by these algorithms, the dots (or the dark spots) in the output have the same size and the gray level effect is obtained by their spatial

arrangement. Algorithms discussed for achieving this spatial arrangement are of two types; one where the resolution of both the input as well as the output image is the same, and second, when the resolution of the output image is higher than the input image.

Halftone imagery is also a bilevel image which gives the subjective effect of a continuum of gray levels. The halftone techniques were developed in the mid 19th century for approximating the gray levels available in the natural imagery. There are three main techniques - lithography, letter press and gavure; and all these produce the gray levels via the presence or absence of opaque ink on a page. In order to represent the continuum of gray levels, high frequency line and dot structures are printed which have their width varied spatially throughout the scene to yield a varying percent reflectance across the page. The end result is that when such images are viewed at normal viewing distances, the dot or line structure is not noticeable but the varying average gray level produces an approximation to a natural scene. Examples of halftone images are the pictures printed in books and newspapers.

The third class of bilevel pictures is the line copy imagery. This imagery is composed of alphanumeric characters, straight line segments and solid areas of a

single gray level. This class of images are essentially made up of only two gray levels but unlike the halftone images, the dots and lines of visible size are used. Examples of this imagery are books, magazines (less their halftone pictures) and maps.

1.1 DISPLAY DEVICES

Many displays are basically bilevel in nature which have individual display cells of the same size arranged in a rectangular array. Individual display cells or sites are either on or off; bright or dark; white or black; reflective or absorptive. Examples of these kinds of displays are : Direct view storage tube (DVST), liquid crystal displays, AC-sustained gas discharge or plasma panels. The algorithms studied are used for display of images on such displays.

These display devices have inherent image storage capability and overcome the disadvantage of refresh displays of CRT. DVST resembles a CRT in that it uses a similar electron gun and a phosphorus coated screen. The difference is that the phosphorus has extremely long persistence and the beam does not write directly on the phosphorus, but on a fine mesh wire-grid coated with dielectric and mounted just behind the screen. A pattern of positive charge is deposited on the grid, and this pattern is transferred to the phosphorus by a continuous flood of electrons issuing

from a separate flood gun. Just behind the storage mesh is a second grid, the collector,whose main aim is to smooth out the flow of flood electrons. These electrons pass through the collector at a low velocity, and are attracted to the positively charged portions of the storage mesh but repelled by the rest. Electrons not repelled by the storage mesh pass right through it and strike the phosphorus. To increase the energy of these slow moving electrons to create a bright picture, the screen is maintained at a high positive potential. Fig. 1.1 shows the general arrangement of the DVST. The outputs of the study have been generated on such a device, the Tektronix 4006-1, which incorporates a 7 by 10 inch DVST and a built in alphanumeric keyboard.

The AC-plasma panel functionally is very similar to the DVST, even though its construction is very different. Images can be written onto the display surface point by point; each point remains bright after it has been intensified. Construction of the plasma panel is shown in Fig. 1.2. It consists of two sheets of glass with thin, closely spaced gold electrodes attached to the inner faces and covered with a dielectric material. The intervening space between the sheets of glass is filled with neon-based gas and sealed. By applying voltages between the electrodes the gas within the panel is made to behave as if it were divided into tiny cells, each one independent of its neighbours. A

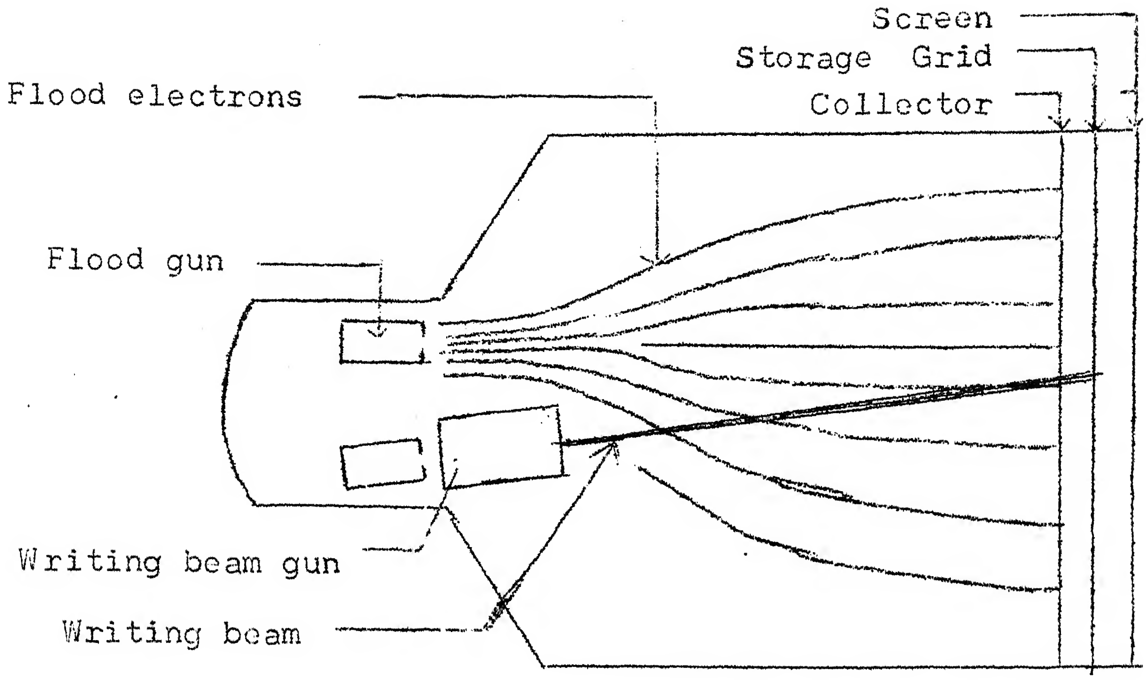


Fig. 1.1 The Direct-View Storage Tube

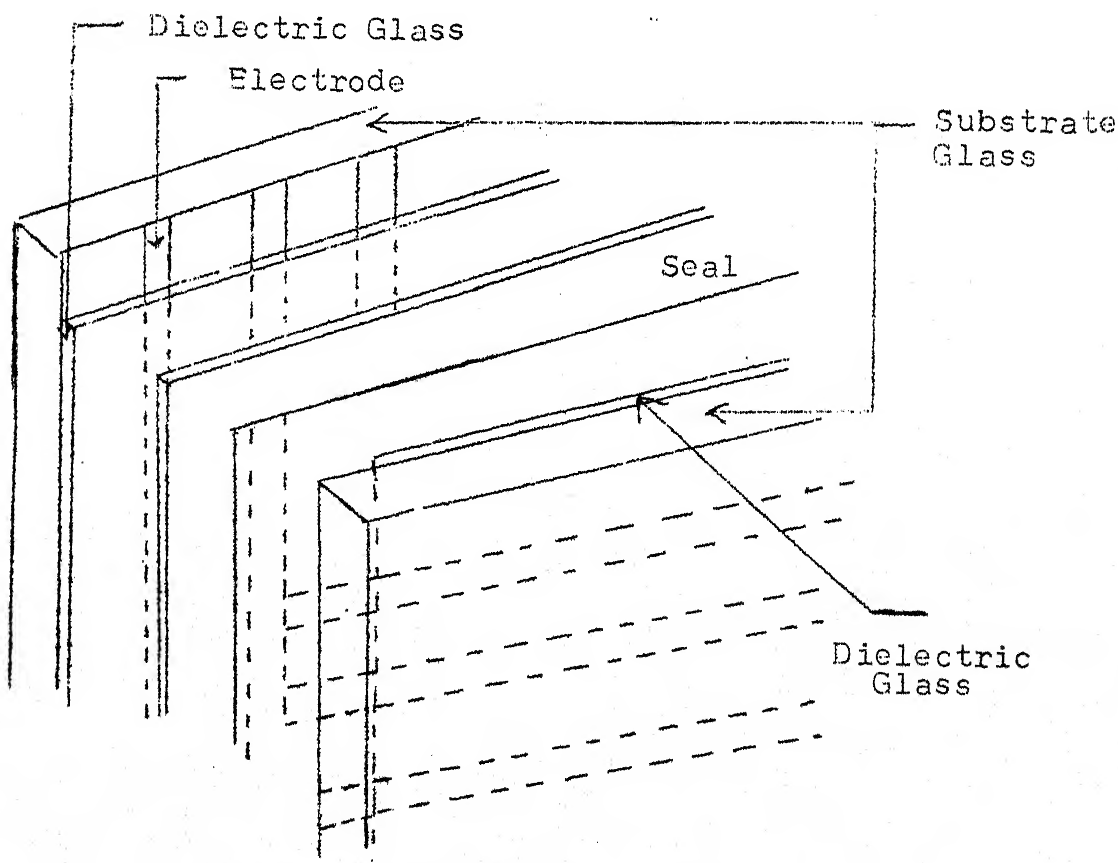


Fig. 1.2 The Plasma panel

cell is made to glow by placing a firing voltage across it by means of electrodes. The gas within the cell begins to discharge, and this develops rapidly into a glow. The glow is sustained by maintaining a high frequency AC voltage across the cell.

Among the newer display techniques, the thermal effects in liquid crystals are made use of to display pictures. Fig. 1.3 shows an arrangement using liquid crystals. It employs a laser beam for writing. Here, all the liquid-crystal molecules are assumed to be initially aligned perpendicular to the glass plates, making the layer appear clear. When scanned with the modulated laser beam the energy absorbed results in local heating. This raises the material above the transition temperature, causing the molecules to become completely disordered. After the removal of the laser beam, the material cools rapidly and the molecules remain randomly oriented, producing a scattering effect which stays for very long. The antireflection layer is provided to increase the absorption of energy from the laser beam. Light which enters clear elements of the liquid crystal is reflected back from the rear aluminium electrode. This light reaches the screen through the aperture. However, the light reflected from the scattered elements is emitted in a variety of directions with a large fraction being unable to pass through the aperture. Since

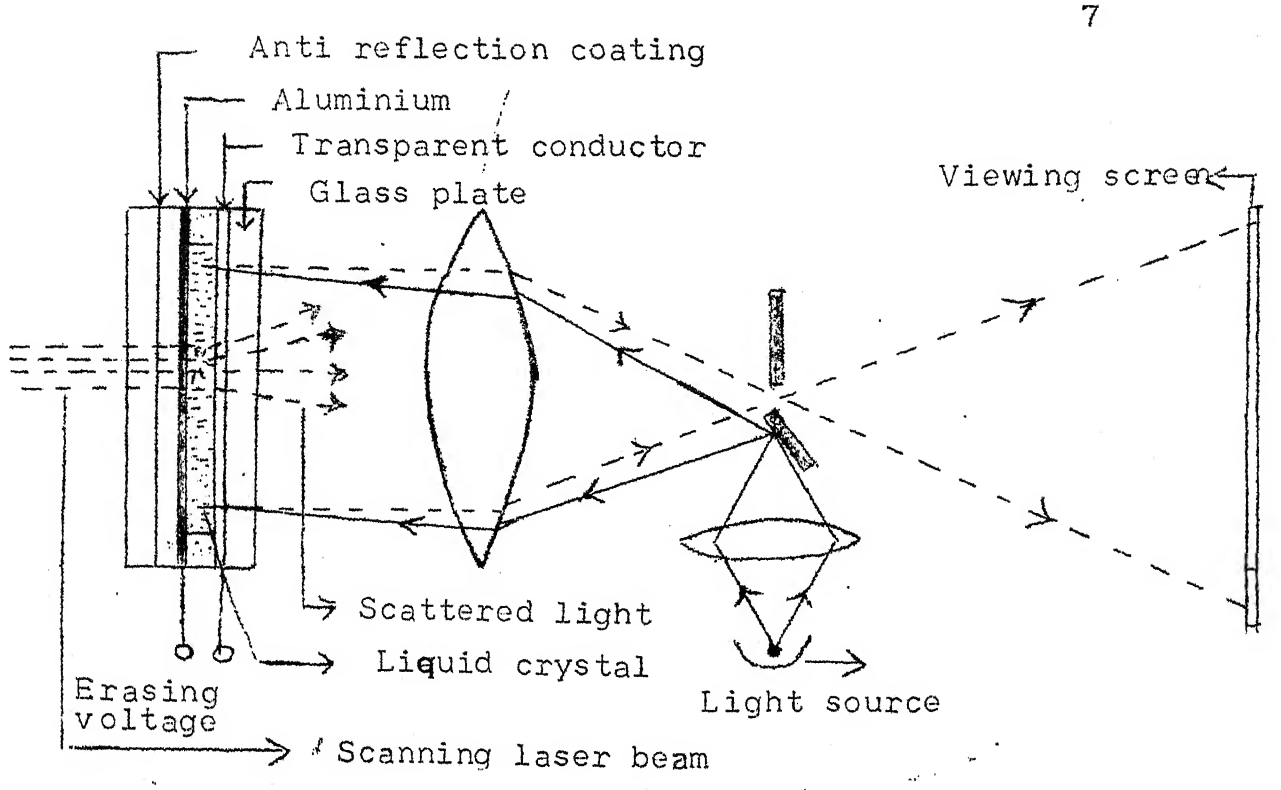
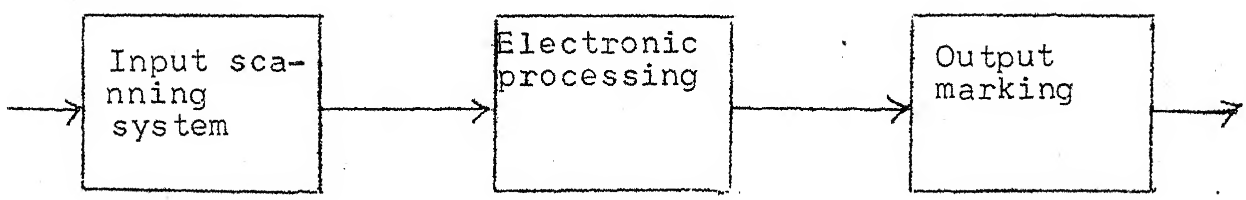


Fig. 1.3 The liquid crystal display



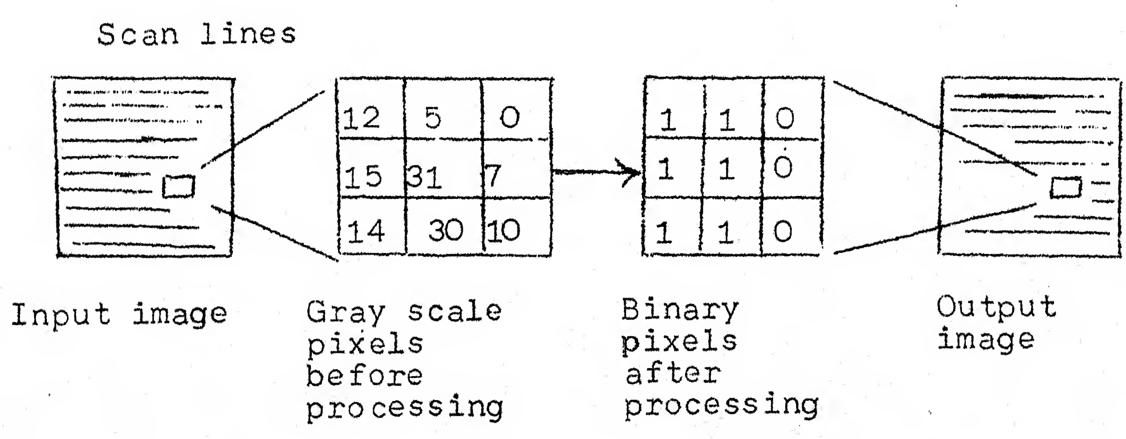


Fig. 1.4 Functional block diagram of bilevel image processing

the lens also serves to image the liquid crystal layer into the viewing screen, elements addressed by the laser beam appear as a pattern of dark spots against a light background on the screen. The above devices are discussed in detail in [1] and [2].

1.2 SCOPE OF THE STUDY

We can model the picture processing problem as given in Fig. 1.4. The input is restricted only to multilevel computer generated and continuous tone pictures and the output is a binary picture. The algorithms are used to process the pictorial input for high fidelity binary output pictures. The processing involves the selection of a threshold value which is used to spatially arrange the dots/blanks at the output.

The study has been carried out with the help of a 32-gray level computer generated picture of Abraham Lincoln. This picture has been chosen from a set of 32-gray level pictures given as 64x64 matrix of sampled intensity values at Appendix 'A' of [3]. The algorithms are applicable to any continuous tone picture whose scanned intensities are available. The processing has been done by simulation on the DEC-10 system and the hard copies of Lincoln's pictures have been obtained with the help of the plotter available with the Tektronix 4001-6 graphics terminal. The pictures generated are of three sizes; 64x64, 128x128 and 256x256.

The available 64x64, 32-level data of Lincoln's picture was processed to generate the 128x128 and 256x256 sized data. The data was processed in two ways : repetitive and random. In the former, each pixel's gray value was replaced by a 2x2 or 4x4 matrix consisting of the same pixel gray value repeated to generate 128x128 and 256x256 sized data respectively. For random processing the data was generated by generating a set of random numbers lying in the interval between the pixel's gray value and the gray average over a 3x3 window. The required number of these random numbers were then chosen and used to replace the original value. The computer programs used for generating these data, DIST.FOR and FDIST.FOR are attached at Appendix B. To generate the random numbers the NAG library subroutine GO5DDF was used.

To evaluate and compare the various algorithms, the correct method perhaps would be to assess the pictorial outputs by subjective tests involving a number of subjects evaluating and responding to the quality of the picture. Since that is a long process and will probably be more suitable for a specific application, three image quality metrics given below were chosen as reasonable components or embodiments of the subjective judgements.

(a) Low Frequency Rendition : Capability of the process to reproduce the low frequency pictorial information in the input.

(b) High frequency Rendition : Capability of the process to reproduce fine details. That is to say that an algorithm capable of reproducing the same contrast as that of the input will be superior to an algorithm which reproduces the same fine detail with lesser or no contrast.

(c) Processing Artifacts : This implies those details in the output picture which are not part of the original picture but have appeared as a result of the processing technique. False contours are a type of artifact and may be caused due to the gray level quantization step which are sufficiently large to create a visible contour when the input image is truly a smooth, gradual variation from one to the other gray level. Another type of artifact which is similar to the false contour is caused by the output patterns. In order to get a gray level with the help of the binary process, called orthographic technique, use of binary patterns over some area is made to provide an average reflectance which is equivalent to a desired gray level. Such output patterns may create certain textures.

Another metric which may be used to compare and evaluate the various algorithms is the complexity of each of these algorithms in implementation. Though the algorithms are not very complex but keeping in view the real time implementation, even small computational difference may also

turn out to be of reasonable importance. The measure utilized is the 'context' required by each algorithm. The context is defined as the size of the memory required to determine the output state of a given pixel. For example, in a single level thresholding (like the fixed thresholding) the decision over the state corresponding to a given pixel is simply made by just that pixel being made available whereas, in case of an algorithm where a window (local) average has to be computed before the state of the output pixel can be decided, the context is the number of pixels required in the computation of the average. As seen, it does not consider operations like number of multiplications etc. to gauge the complexity.

No study of image processing is complete without reference to the capability of the human visual system as all the pictures are ultimately viewed by humans only. Towards this end the perception by the eye is of specific importance as the eye sees the gray level in the binary picture due to its inherent property of spatial integration. Efforts have been made to approximate the eye by a mathematical model. One such model has been chosen from the various models available in the literature and the processing has been done for a set of 3x3 binary patterns as inputs. The output 2-dimensional arrays give the mathematical approximations to what the eye sees. Some of these outputs have been plotted with the help of 2-dimensional hidden line plots to give the

effect of a surface seen by the eye. This aspect has been covered in Chapter II. Subsequently sets of 4x4 and 6x6 binary patterns have been chosen to output the Lincoln's picture by the orthographic technique. The resulting picture is a 256x256 and 384x384 pixel picture respectively as for each pixel of the input picture we print a 4x4 or 6x6 binary pattern at the output.

Other techniques discussed differ from the orthographic technique in that they have the same resolution at the input and output. Eye's perception of gray levels in such pictures is spread over a small area to give a general effect of an average gray level.

To facilitate comparison between the various algorithms, the following notations have been used. A frame is defined as a set of N^2 samples taken on a square geometric grid of N lines with N samples per line. In this study, three different values of N (64, 128 and 256) have been considered. Each of the N^2 samples is the intensity value $I(x,y)$ of the point (x,y) in the picture, subscripts x and y fixing the position within a line and within a frame. Since the Lincoln's picture is a 32 level picture, $I(x,y) = 0$ implies the darkest gray level and $I(x,y) = 31$ means the brightest gray level. $O(x,y)$ denotes the displayed intensity which is either 0 or 31 as the picture displayed either has a dot or a blank.

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CHAPTER II

THE EYE AND ITS MODELS

Light as per Webster's dictionary is : 'radiant energy which by its action on the organs of vision, enables them to perform their function of vision'. Light as we know is a form of electromagnetic radiation lying in the relatively narrow region of the electromagnetic spectrum over a wavelength band of about 350 to 780 nm. The mechanisms by which the light interacts with the 'organs of vision' is not very well understood. However, an attempt is made in this chapter to review some of the mathematical models of the eye available in the literature and then one of them is chosen and used to show analytically how the eye perceives a fine binary pattern structure(of dots and blanks) as different gray levels.

2.1 THE EYE

A conceptual technique for the establishment of a model of the human visual system (HVS) would be to perform a physiological analysis of the eye, the wave paths to the brain, and those parts of the brain involved in visual perception. Such a task, of course, is beyond the ability of the man because of the large number of infinitesimally small elements in the visual chain and their complex interconnections. Fig. 2.1 contains a sketch of the horizontal

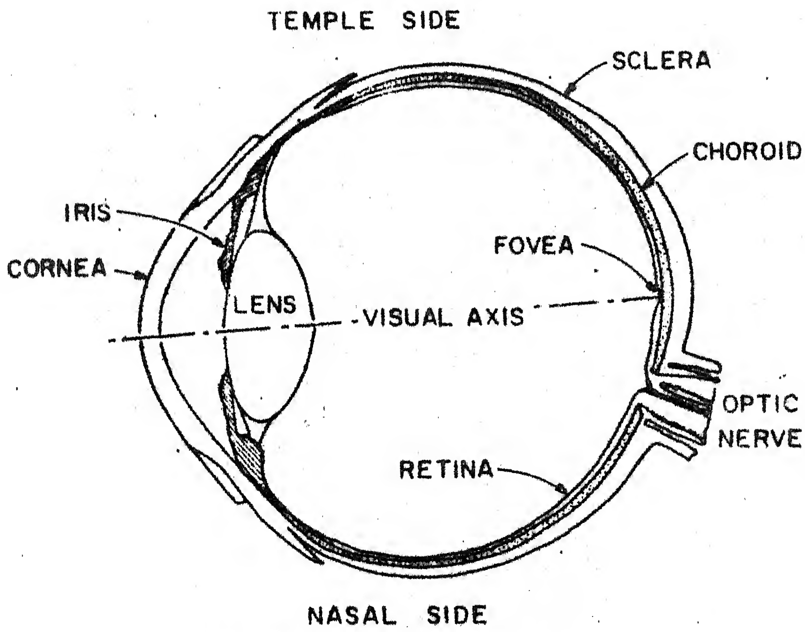

FIGURE 2.1 **Eye cross section.**

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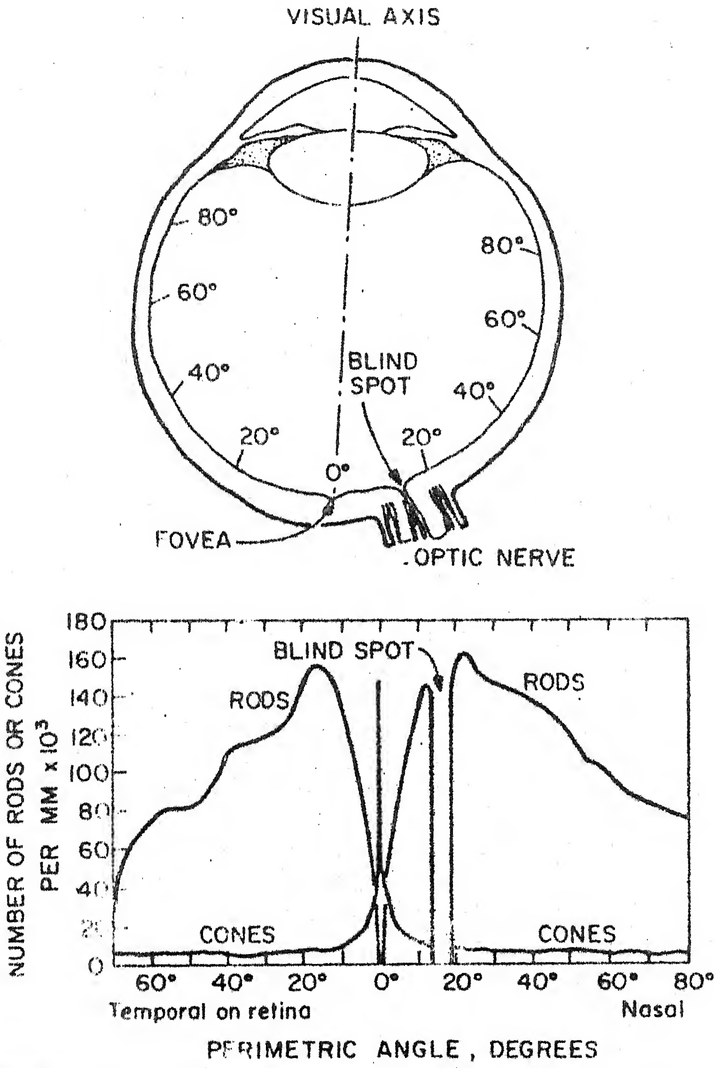

FIGURE 2.2 Distribution of rods and cones on retina

cross-section of the human eyeball. The front of the eye is covered by a transparent surface called the cornea. The remaining outer cover called the scalera, is composed of a fibrous coat that surrounds the choroid,a layer containing blood capillaries. Inside the choroid is the retina, which is composed of two type of receptors - rods and cones. Nerves connecting to the retina leave the eyeball through the optic nerve bundle. Light entering the cornea is focussed to the retinal surface by a lens that changes shape under muscular control to perform proper focussing of near and distant objects. An iris acts as a diaphragm to control the amount of light entering the eye. The rods in the retina are long slender receptors, while the cones are generally shorter and thicker in structure. Operationally, the rods are more sensitive than cones to light. At low levels of illumination, the rods provide a visual response called scotopic vision. Cones respond to higher levels of illumination; their response is called photopic vision. An eye contains about 6.5 million cones and 100 million rods distributed over the retina. Fig. 2.2 shows the distribution of rods and cones over a horizontal line on the retina. At a point near the optic nerve called the fovea, the density of cones is greatest. This is the region of sharpest photopic vision. There are neither rods nor cones in the vicinity of the optic nerve, and hence the eye has a 'blind spot' in this region. When a light

stimulus activates a rod or cone, a photochemical transition occurs producing a nerve impulse. The manner in which the nerve impulses propagate through the visual system is presently not well established. It is known that the optic nerve bundle contains on the order of 800,000 nerve fibres. Since there are 100,000,000 (100 million) receptors in the retina, it is obvious that in many regions of the retina, the rods and cones must be interconnected to nerve fibres. Since, neither the photochemistry nor the propagation of nerve impulses within the eye is well understood, a deterministic characterization of the visual process is not available. However, certain models of the HVS have been presented which are taken to predict the human visual system response.

Before the HVS model is discussed, certain visual phenomena that a reasonable model must accomodate are discussed :

a) Dark adaptation of the eye :

It is well known that a person is more sensitive to dim flashes of light if he has been in dark for a period of time (as happens when one enters a movie hall). Cornsweet [1] has shown that the cones are adapted to dark in about 5 minutes whereas the rods take much longer (about 30 minutes).

b) Threshold intensity for perception :

It is the lowest possible intensity that the eye can see. It is dependant upon the colour, location with respect to the eye, timing and size of the light. The state of the subjects eye will also affect the threshold value. Any light lost before it reaches the rods (being the primary receptors at low intensities), cannot affect the subject as he does not assimilate any information about the light if it is not absorbed by the visual pigment in the rods. It has been shown by Cornsweet (p.25) that after catering for all the losses of light like reflection from the eye, absorption by the pigment that fill the eyeball itself, and the loss due to the inter rod spacings; about 10 quanta are absorbed, but they are spaced over an area which has 500 rods. Probability of one rod absorbing two quanta of light will be very small and hence one quantum is sufficient to stimulate a single rod and approximately 10 rods within a 10 µm diameter area must be stimulated within 1 msec of each other before the stimulus is perceived. Hence, the activation of 10 rods is somehow added up in the visual system. This threshold is for monocular vision. The binocular threshold is lower by a factor of $\sqrt{2}$ as mentioned by Hall [2].

c) Simultaneous contrast

It is commonly experienced that the brightness of a region does not depend upon its intensity only but also upon

the surround. If we look at a thick (opaque) gray or coloured paper with one eye under any ordinary light and then hold it up at an arm's length between the eye and a bright sky or a large light source, the paper will look much darker when it is in front of the light source, even though the actual amount of light reflected from the paper is the same. In fact, if the background light is bright enough the paper will look black when held in front of it.

d) Brightness constancy :

It implies that the brightness of any object remains fairly constant despite large changes in the illumination that falls on it. For example, a piece of ordinary white paper reflects about 99 percent of the incident light, while black papers reflects about 10 percent. When these papers are moved from ordinary room light illumination into direct sunlight, the incident illumination and therefore, the reflected light intensity may increase by 1000 fold; yet the papers still have about the same brightness as they had indoors. Furthermore, the black paper outside reflects 0.10x1000 = 100 relative units of intensity and the white papers inside reflects only 0.9x1 = 0.9 units; but the black paper outside still looks darker than the white paper inside.

e) Mach bands :

Consider a set of gray scale strips, each having the same gray level, the darkest strip towards the left extreme and the brightest towards the right extreme. The reflected light from each strip is uniform over its width and differs from its neighbours by a constant amount. Nevertheless, the visual appearance is that each strip is darker at its right side than at its left side. This is called the Mach Band Effect. The effect is due to the spatial frequency response of the eye, which possesses a lower sensitivity to high and low frequencies than mid-frequencies.

2.2 MODULATION TRANSFER FUNCTION (MTF)

In order to postulate a model for HVS, it is necessary to have a method which can be used to predict the response of the HVS given a particular input. Since our concern in this study is the spatial response of the eye, the processing model is based upon the amenable frequency measurements.

Since any spatial pattern can be broken down into various sine wave components by Fourier Analysis, the procedure to measure the MTF is to feed a sine wave grating as the input. Cornsweet [1] has explained this method with the help of an optical lens to which sine wave gratings of different frequencies are input and the outputs are studied to calculate the MTF of the lens. It is seen that as the

frequency of the gratings is increased, the amplitude of the modulation of the image distribution (i.e., the difference between the intensity at each bright part and at each dark part) becomes smaller and smaller and for extremely fine gratings, the modulation in the image becomes zero and a uniform intensity distribution is seen. The property of the lens being measured is its ability to transfer spatial modulation of intensity from the object to the image. Hence, the MTF of the lens is an indication of its ability to resolve gratings. In the experiment above, the phase shifts have been assumed to be zero. If say, the phase shift introduced by the lens was constant for all the frequencies, then the image would merely be shifted from the expected position. However, if the phase shifts vary with frequency then the MTF alone would not be sufficient to define the image of any object as the relative positions of the various sine wave components would not be known. Therefore, if there were substantial phase shifts varying with frequency, both the MTF and the phase shift at each frequency would have to be known in order to predict the image correctly. However, the phase shifts produced by most lenses and by the optical system of the human eye are relatively small near the optic axis (fovea for the eye) and only trivial errors are introduced by assuming zero phase shifts.

Before we use the concept of MTF for the eye model, it is necessary and essential to list out the conditions that must be met in order to use Fourier techniques.

a) Linearity :

A linear system is one that obeys the principle of superposition, i.e., if two inputs give the outputs X and Y, then when both the inputs are applied simultaneously, the system output should be X+Y. In general, if the intensity radiated from the object is increased, the magnitude of the response of the system should increase proportionately. We may also apply the technique over the linear operating region of a nonlinear system.

b) Homogenity :

A system is spatially homogeneous if its characteristics are the same in all locations. If a system is homogeneous, then shifts in the location of the input pattern may cause shifts in the location of the output pattern, but the output pattern will not change except in position. The HVS is not homogeneous due to the variation of the densities of rods and cones with retinal position. However, it is relatively homogeneous near the optic axis (i.e. near the fovea). In spite of the inhomogenity of the HVS, Fourier techniques may be used to correctly predict its response. Cornsweet [1] has pointed out that there is probably a self homogenising process in the structurally anatomically

inhomogeneous process which renders it open for the use of Fourier techniques. Models discussed do not consider temporal responses, hence temporal homogenity has not been considered.

c) Isotropic:

A system is isotropic if its characteristics are the same in all directions. For example, a system is isotropic if the MTF measured with sinusoidal grating objects is the same regardless of whether the gratings are vertical, horizontal or oblique, (i.e. regardless of their angular orientation with respect to the optic axis in the object plane). The response of the HVS to a rotated contrast grating is a function of frequency of the grating as well as the angle of orientation. The sensitivity of the system decreases to a minimum at 45° and then rises again reaching the original level at 90°. At the point of maximum deviation (i.e. 45°) a frequency of 30 cycles/degree would be -3 dB below the response at 0° rotation. Whereas for 10 cycles/degree the response at 45° deviation is only 15 percent lower than the original. This an-isotropic behaviour has not been included in the models discussed.

2.3 THE EYE MODEL

In the lens experiment explained in Section 2.2, the output to a particular input is visible and the transfer function can be calculated in a straight-forward manner. The

case is different for the eye, where we have no way of knowing what kind of output image is being formed in the human brain. Hence, in order to measure the modulation transfer function for the HVS, various ways have been used. But the basic principle used in all is that a subject is asked to match a variable object (grating) with a reference object (grating) and this can be done in different ways like varying the contrast or frequency.

As a direct analogy with the procedure for measuring the MTF of a lens, a human subject could be presented with a series of sine wave gratings with same amplitudes but different frequencies and then obtaining a measure of his perception of the patterns. The measure of the perception is the apparent amplitude of each sine wave, (that is, the difference between the brightness of peaks and troughs of the wave) because the apparent amplitude is exactly an 'output' of the HVS. Thus, a plot of the input frequency (for a fixed amplitude) against the apparent amplitude, would be analogous to the plot of the MTF of the lens. Given such a plot and assuming that the HVS is operating in the linear region, and that it is homogeneous and isotropic, we should be able to predict the appearance of any input pattern. Davidson [1, p. 335] has suggested one such arrangement. Assuming the HVS to be linear, homogeneous, isotropic, monocular, monochromatic and photopic (that is operating beyond threshold), a MTF was

plotted and is given in Fig. 2.3. In fact, the response is contrast dependant and the curves for different ratios of contrast have same shape with higher ratio curves, higher in amplitude. This response can be represented by a system which is just a band pass filter but it would be difficult to defend from a physiological standpoint since the response is a compound one due to several mechanisms within the HVS.

A slightly more detailed model would be a combination of a LPF and a HPF. In fact, the complete response of the HVS as in Fig. 2.3 can be depicted by a variable sine wave grating of Fig. 2.4. The figure shows sine wave gratings with frequency increasing from left to right and the contrast increasing from bottom to top. If a line is drawn horizontally across the figure, the line will intersect sinusoidal changes in intensity whose amplitude is constant but whose frequency varies. As can be seen clearly from the diagram, sensitivity of the HVS is maximum at mid frequencies and reduces at lower and higher spatial frequencies. Cornsweet (p. 343) has used this MTF to explain the Mach band effect and contrast sensitivity mentioned earlier in this chapter.

Characteristics of the high frequency response, i.e., the region that extends from peak **mid frequency** where the sensitivity decreases with frequency,

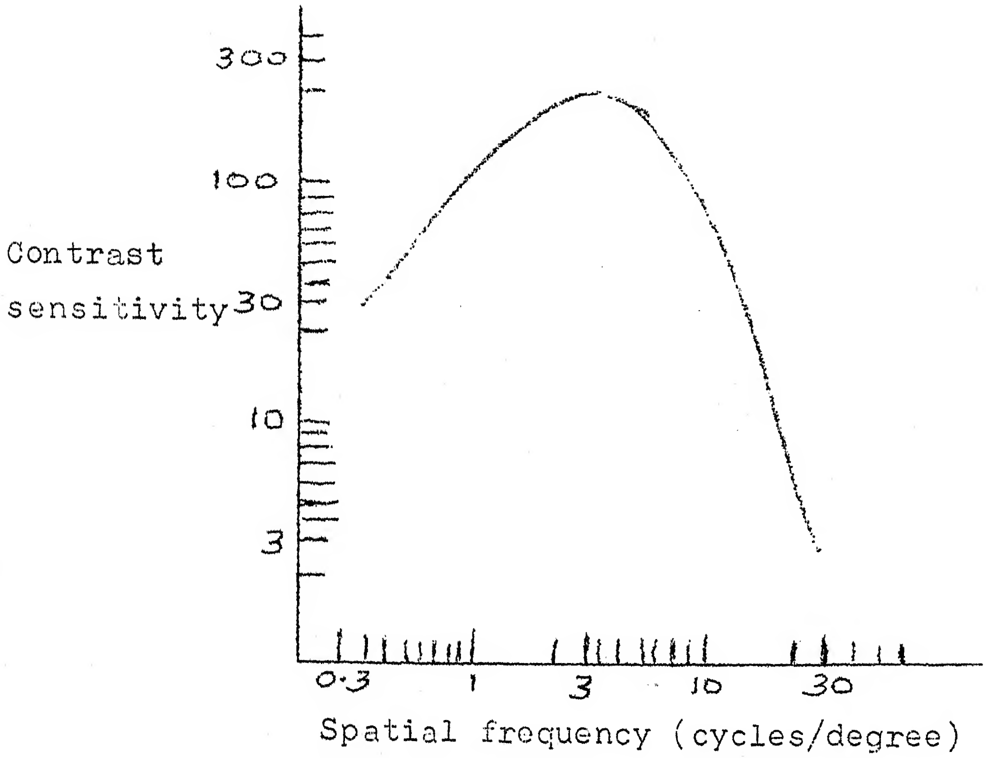

Fig. 2.3 Response of the Human Visual System

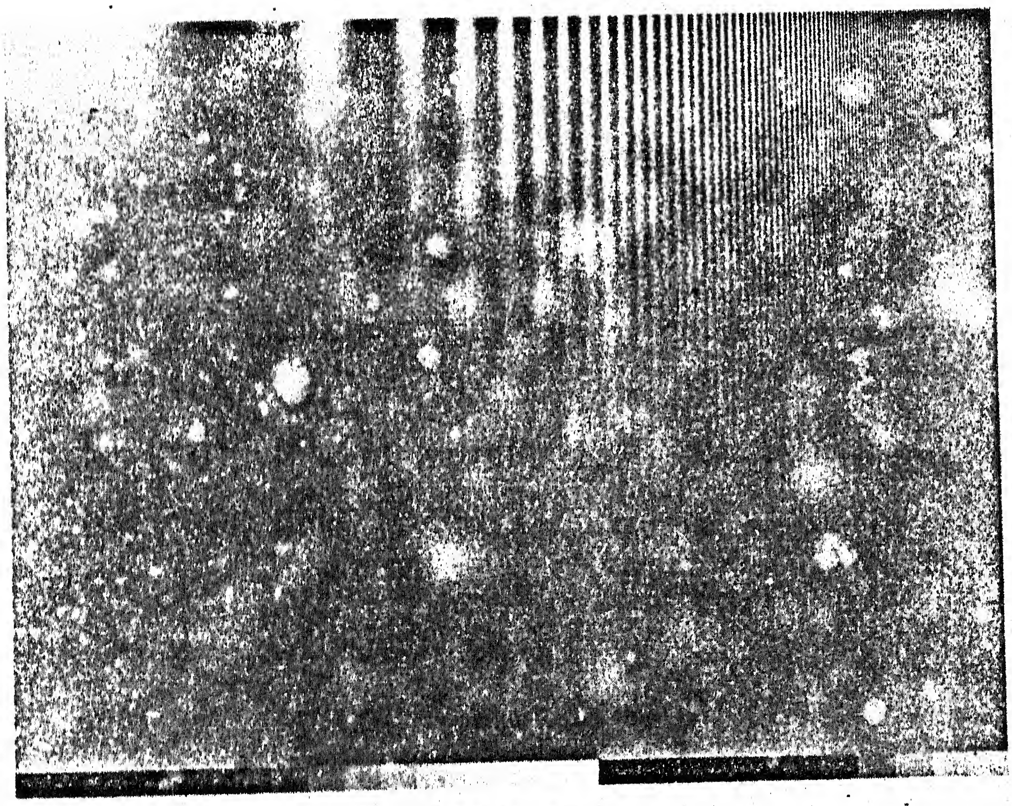

FIGURE 2·4 **MTF measurement of human visual process by modulated sine wave grating.**

are determined in part by the optics of the eye (lens, pupil size and cornea); also the properties of the retina itself affect the high frequency response. Size and density of the photo receptors, neural summation and scattering of light at the retinal surface affect the high frequency response of the HVS. The low frequency response is limited by the lateral inhibition.

This model of a combination of LPF and HPF can be used to approximate a HVS but it does not account for the phenomenon of 'brightness constancy'. This phenomenon can be accounted for by introducing a nonlinearity in the system. This is illustrated in Fig. 2.5(a) where the intensities in the right hand figure are all the same multiple of those on the left. If a logarithmic transformation is assumed, the figures get transformed as in Fig. 2.5(b). The two outputs now look quite similar which is how the eye perceives the black and white papers in normal illumination and under bright illumination source. That is, if a logarithmic transformation is added to the MTF described earlier, we are in a position to accomodate in the model the majority of the phenomena of the eye's perception of still, monochrome images. This nonlinearity has been studied extensively by various researchers. The studies were mainly carried out on the horseshoe crab, 'Limulus', and with the use of mechanical receptors. The choice of Limulus was motivated because

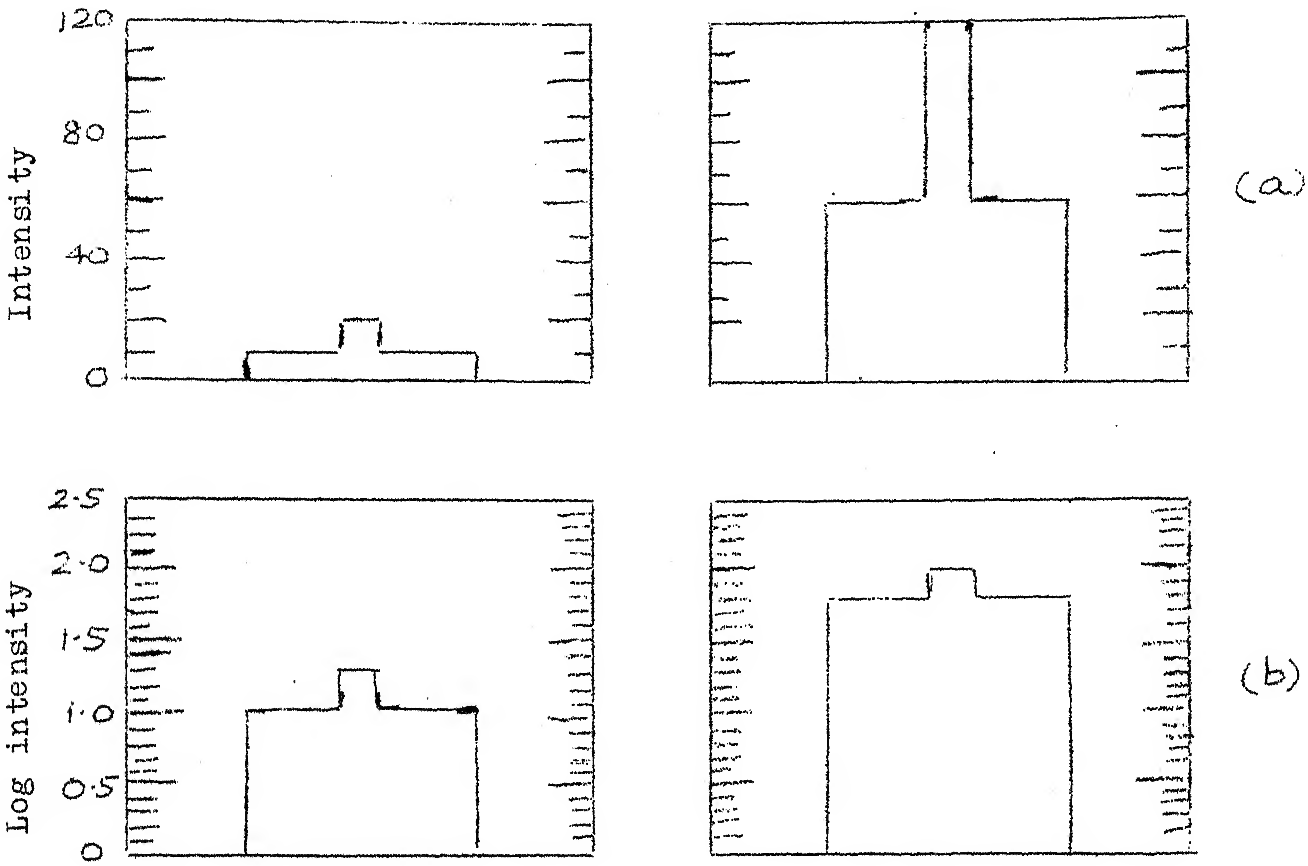

Fig. 2.5 Patterns for brightness constancy

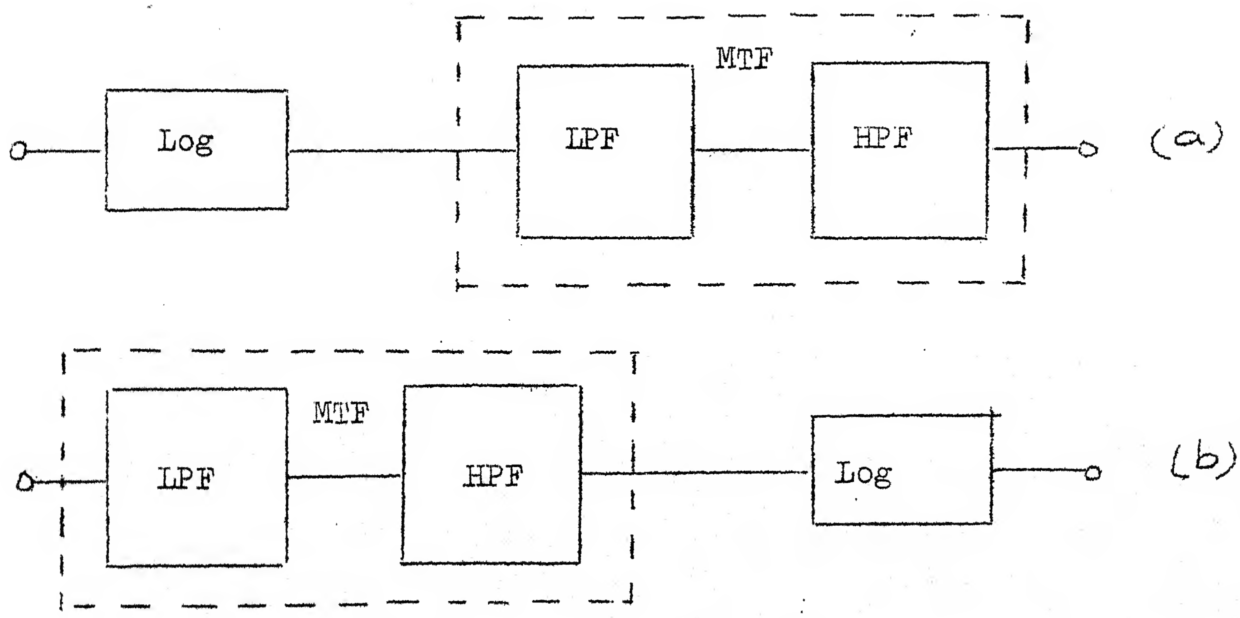

Fig. 2.6 Nonlinear models of HVS

the lateral eyes of this animal lend themselves to physiological analysis, and preliminary studies suggested that many of the properties of this eye are similar to those of higher animals, including humans. It has been shown that the nonlinearity in the HVS can indeed be approximated by a logarithmic function.

The question which arises is how a nonlinearity can be added to MTF, (a linear system). It has been argued by Cornsweet (p. 334), that for small variations in intensity of input patterns, the system essentially operates in the linear region of the nonlinearity. Also, it has been shown that under increasing light adaptation condition; the linear region becomes larger.

Two ways in which the nonlinearity can be added are shown in Fig. 2.6. Of the two structures shown Fig. 2.6(a) can be used to explain the phenomenon of brightness constancy. This system is thus physiologically sound and it also predicts the brightness constancy. This model has been chosen to process the 3x3 binary patterns. The impulse response of the LPF has been given by Campbell [3] as :

$$h_1(x) = \exp(-\alpha \, |x|) \qquad (2.1)$$

where α has a value of 0.7 for a pupil diameter of 3 mm, Its Fourier transform is given by

$$H_1(\ ) = \frac{2\alpha}{\alpha^2 + \omega^2} \qquad (2.2)$$

The −3 dB point occurs at approximately 6.5 cycles/degree. The transfer function os HPF is given as :

$$H_2(\omega) = \frac{a^2 + \omega^2}{2a_o a + (1-a_o)(a^2 + \omega^2)} \tag{2.3}$$

where a_o is the distance factor between receptors and 'a' is the inhibiting factor. This transfer function has been derived by using the backward inhibition model for the photoreceptors in HVS given in Fig. 2.7. The receptors are assumed to have a logarithmic response to the incoming light. The principle used is that there is an inhibiting effect in between the receptors in the retina. The detailed analysis has been done in the work by Hall and Hall [2].

2.4 RESPONSE TO BINARY PATTERNS

After having discussed a model for the HVS, the next step is to use it to analyse its response to a set of patterns of binary pixels. As was mentioned briefly in Chapter I, due to the spatial integration properties of the eye, a set of properly chosen closely spaced binary patterns with their basic repetition frequency beyond the eye's cutoff range are perceived as different levels of brightness. This property is usefully utilized to transform multilevel computer generated (or continuous tone) pictures into binary pictures which retain the gray level effects due to the spatial arrangement of the binary patterns.

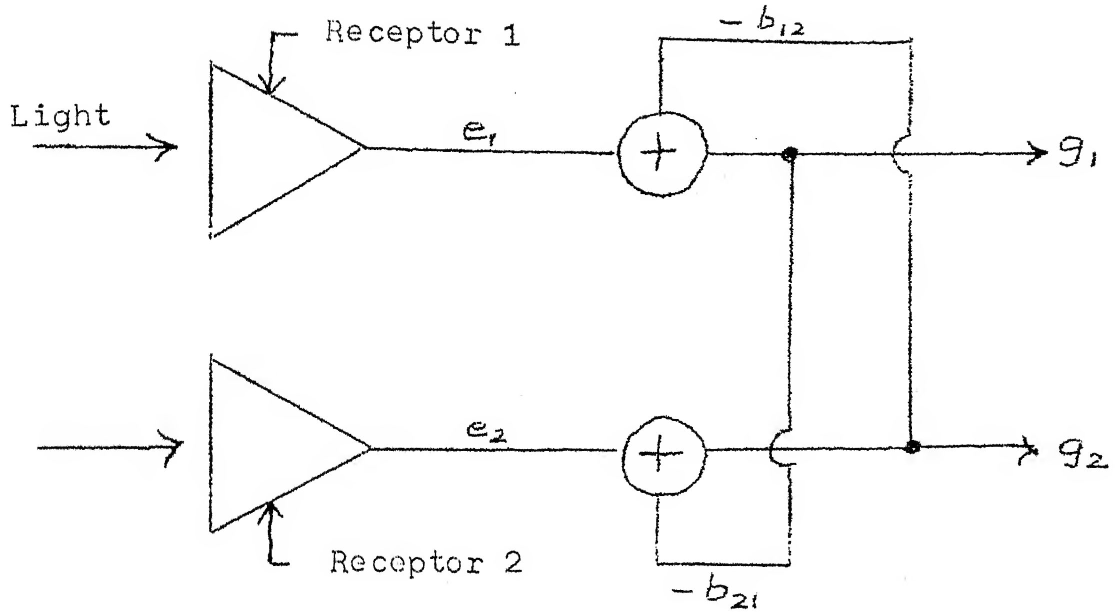

Fig. 2.7 Backward inhibition model

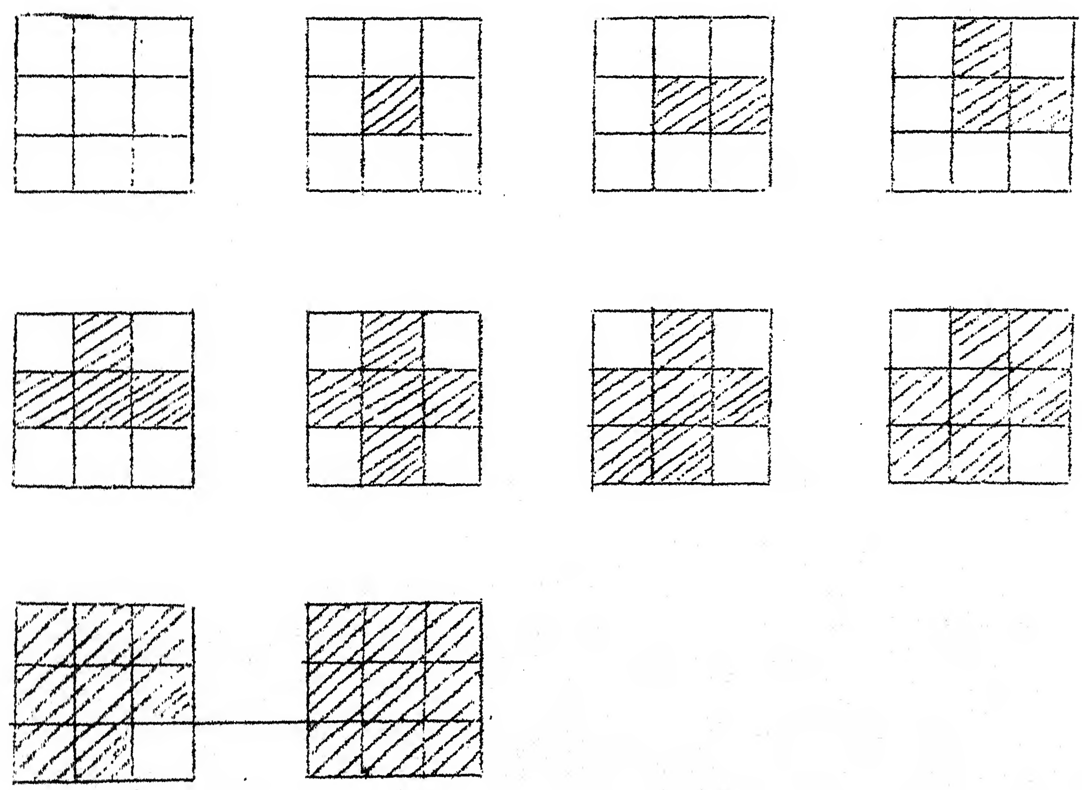

Fig. 2.8 Set of 3x3 patterns

A set of 3x3 such binary patterns which can be used to display 10 intensities are given in Fig. 2.8. This set of binary patterns has been taken from the book by Newman and Sproull [4]. These patterns are input to the model of the HVS and the outputs obtained with the help of the computer program PROC·FOR.

To find out the MTF, the impulse response of the LPF is convolved with the response of the HPF to give the overall impulse response of the model less the nonlinearity. The impulse response of the HPF is worked out as :

$$h_2(x) = \frac{C}{2D} \exp(-D|x|) - \frac{1}{2DA} [D^2 \exp(-D|x|) - 2D\, \delta(x)] \quad (2.4)$$

where

$$A = (1-a_o) = 0.8$$

$$B = \frac{2a_o a}{1-a_o} = 0.005$$

$$C = \frac{a^2}{A} = 0.000125$$

$$D = (B+a^2)^{\frac{1}{2}} = 0.0714$$

On substituting these values, we get

$$h_2(x) = 1.25\, \delta(x) - 0.04375 \exp(-0.0714\, |x|) \quad (2.5)$$

Hence, the overall response of the system is

$$h(x) = h_1(x) \circledast h_2(x) = 1.2531 \exp(-0.7\, |x|) \\ -0.06125 \exp(-0.0714\, |x|) \quad (2.6)$$

Assuming circular symmetry, the two-dimensional response of the system is

$$h(x) = 1.2531 \exp(-0.7\ |x+y|) - 0.06125 \exp(-0.0714\ |x+y|) \qquad (2.7)$$

The input to the eye model is one of the 3x3 binary patterns in which the dot is assigned value 2 and blank is assigned value 11. The pattern has been transformed into a two-dimensional array with the centre pixel being the origin. In order to find the response of the system, we consider only the discrete values of the system point spread function at the points at which the patterns exist. The systems response has been limited to a size of 21x21 as beyond this region the values of the output are quite small. The system response thus is the two dimensional convolution of the log of the 3x3 pattern and the MTF. The two dimensional convolution has been tried out in two ways : the usual double summation and a method suggested by MacAdam [5] to convert the 2-dimensional convolution into 1-dimensional convolution. In this method the 2-dimensional arrays are read as vectors by the computer and then 1-dimensional convolution is performed to yield a vector which is rewritten as a 2-dimensional array to give the result. Though while using the DEC-10 system the difference in the CPU time was negligible as the arrays were not large.

The computer program used to carry out this operation is named CONVOL·FOR and is attached at Appendix B. The program generates the 21x21 matrix which is the MTF of the system. The input to the program is the 3x3 binary pattern. The output is a 23x23 two dimensional array which has the centre value i.e., the 12th column of the 12th row maximum. These values progressively decrease and are minimum at the four corners of the array. As seen from Appx.C for t 10 patterns (and their combinations) the centre value is highest for the brightest level, i.e., the pattern having all bright squares and is lowest for the pattern having all dots, thus the 10 gray levels are distinguished as different from each other. To get a better idea of these outputs, some of these have been plotted with the help of 3-dimensional plots with and without hidden line elimination and are shown as Figs. 2.9 and 2.10.

The 3-dimensional plotting without hidden line elimination is done by fitting a curve (using cubic spline fit) for the values generated by CONVOL·FOR. The algorithm used for this purpose has been suggested by Cline [6]. Cline has suggested curve fitting by cubic splines with tension. The tension factor removes the defect of inflection points commonly seen in the cubic spline fitted curves. With the tension factor, the curve may be assumed to be a light and flexible bar not only constrained to pass through the given values but also able to respond to a tension produced by pulling on its ends. Sufficient tension would remove the

unwanted inflection points. The program consists of the driver program SPLINE·FOR and two subroutines KURV1 and KURV2. These are attached at Appendix B. The plot drawn is shown at Fig. 2.9. This does not incorporate the hidden line elimination. The curves have been rotated by 20° to give the 3-dimensional effect.

The plotting with hidden line elimination is carried out with the help of an algorithm suggested by Watkins [7]. This algorithm incorporates hidden line elimination and the plots are rotated to give the desired 3-dimensional effect. The input to this program is the output of the SPLINE·FOR program. Program PLOT3D·FOR accepts the three dimensional data, rotates it in the three space and plots the projection of the resulting figure onto the X-Y plane. One point to note here is that the rotation carried out by PLOT3D·FOR is not in addition to the rotation carried out by SPLINE·FOR because the data output by SPLINE·FOR is in unrotated form and the rotation that was achieved was with the help of the GPGS package while drawing the plot on the screen.

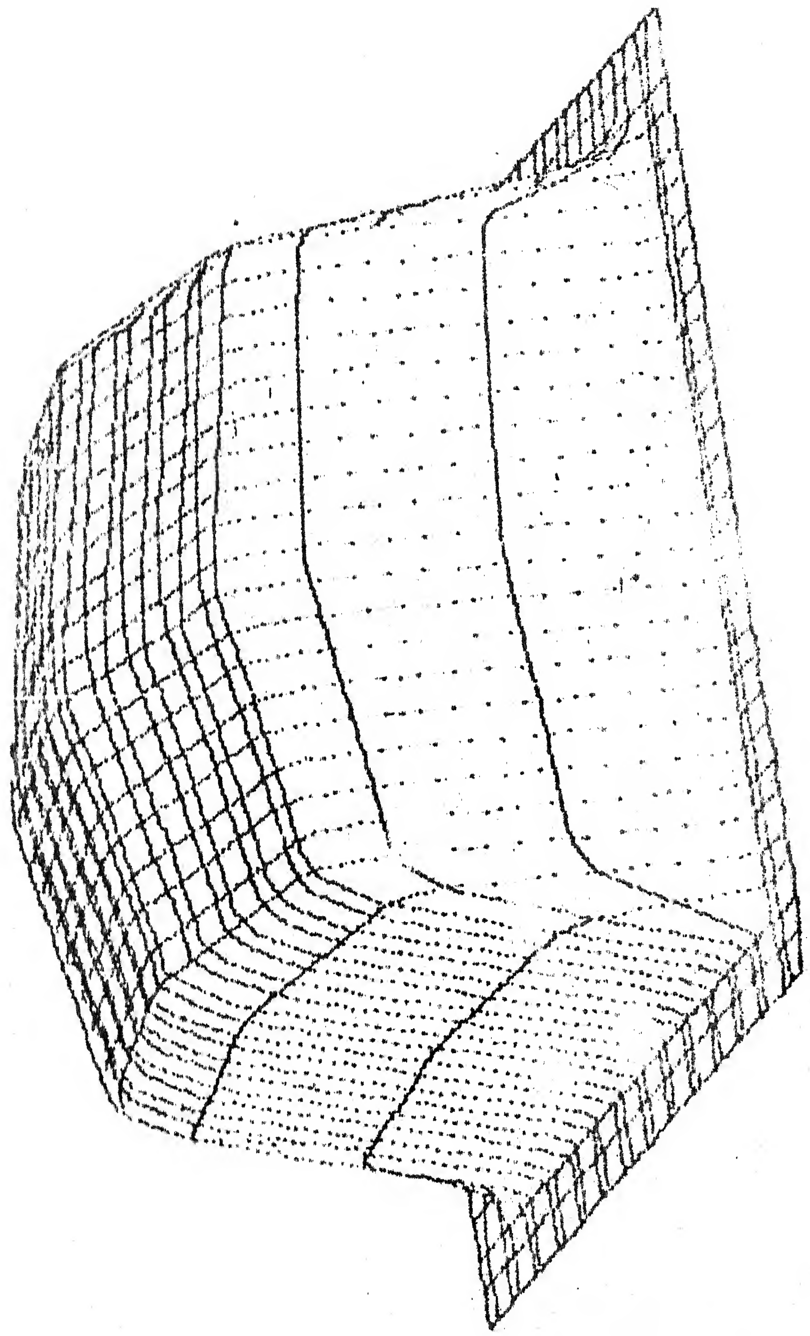

Fig 2.9

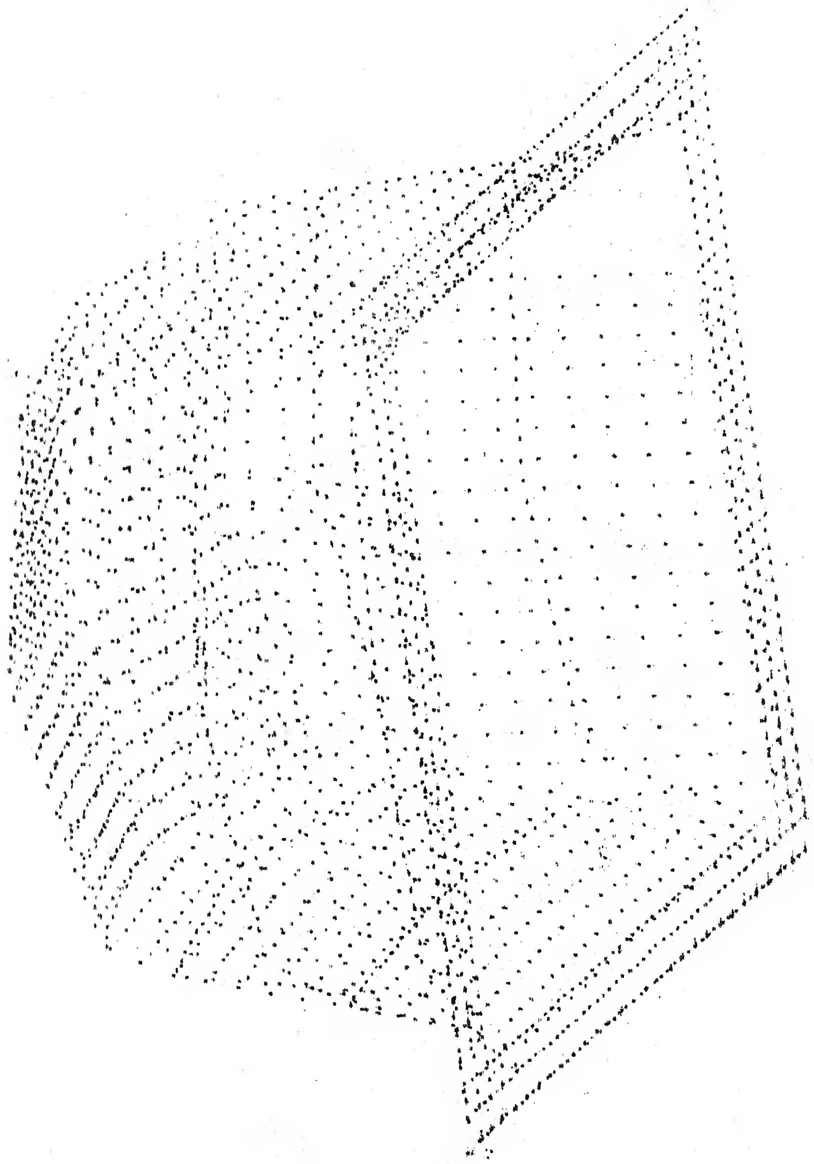

Fig 2.10

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CHAPTER III

TECHNIQUES FOR DISPLAY OF MULTILEVEL GRAY PICTURES ON BINARY DISPLAYS

3.1 INTRODUCTION

As mentioned in Chapter I, there are two classes of techniques to display multi-gray level (continuous tone or computer generated) pictures on binary displays. They differ in the relationship between the number of pixels of the input and output pictures. Gray levels were approximated by patterns of binary pixels in the previous chapter and the technique using this principle, called the Orthographic Technique, has higher number of pixels in the output picture as compared to the pixels in the input picture. The remaining techniques discussed in this chapter belong to the general class of algorithms that convey gray level information by the spatial arrangement of binary pixels which are in a one to one correspondence with the pixels in the original image. These techniques create patterns of binary pixels that preserve detail and average image brightness over extended areas.

3.2 ORTHOGRAPHIC TECHNIQUE

This technique utilizes an 'mxn' array of binary pixels in the form of a gray scale 'character' to represent a multi-gray level picture. These characters together form a

gray scale 'font' which, when printed with minimal inter-character spacing, yield the gray scale information of the original picture.

In Chapter II a set of 3x3 binary patterns were shown and they could be used to represent a 10 gray level picture. Similarly, Hamill [1] has suggested a set of 4x4 binary pixel patterns depicting 16 gray levels. This set of patterns has been used to generate the test picture of Lincoln shown in Fig. 1 of Appendix A. The generated picture is shown at Fig.2a of Appendix A. The program used for the technique is called ORTHO·FOR and is attached at Appendix B. The algorithm simply involves a table look up process as shown in Fig. 3.1. Since the patterns number only 16, a quantization step of two has been used to represent the 32 gray levels. The program ORTHO·FOR reads the patterns as vectors of 16 and then arranges them as 4x4 patterns at the output for each input pixel. The picture generated is thus a 256x256 pixel picture.

Since the usual pictures are sampled with the help of 128, 256 or even higher gray levels, a compromise is usually developed for the size of the patterns as large sized patterns produce coarse resolution. In view of this, large quantization steps will be needed which result in tone scale errors. Because of the coarseness of tone scales, false contouring might result and the actual characters used to create gray

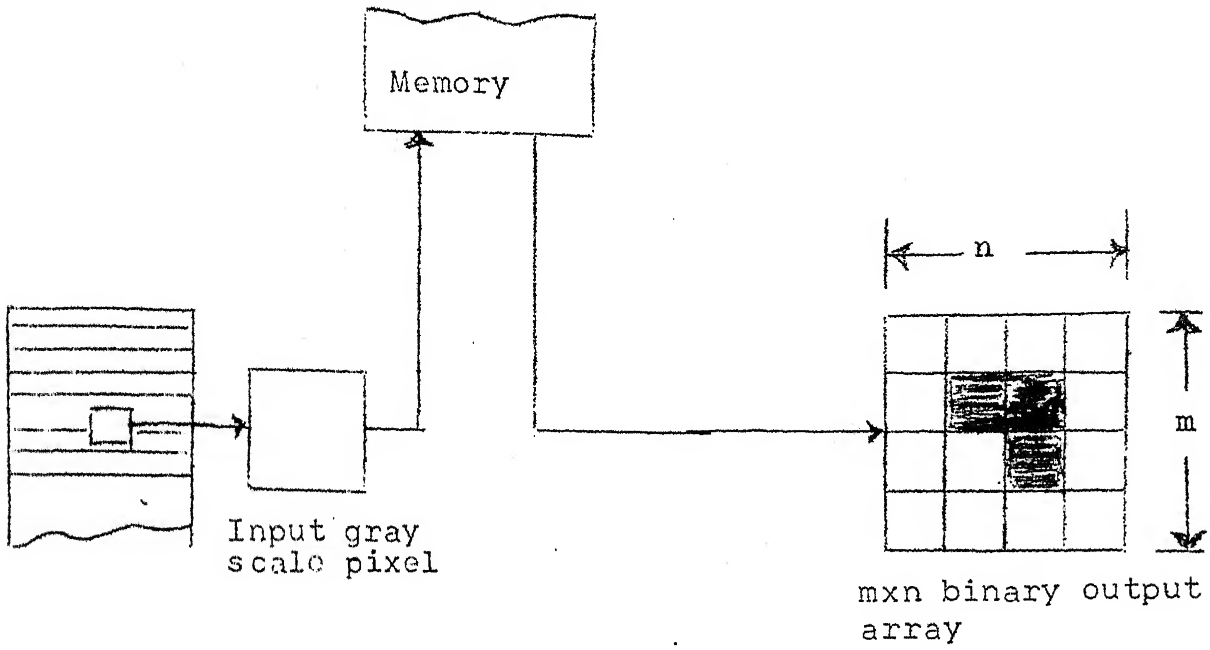

Fig. 3.1 Signal flow diagram for Orthographic Technique

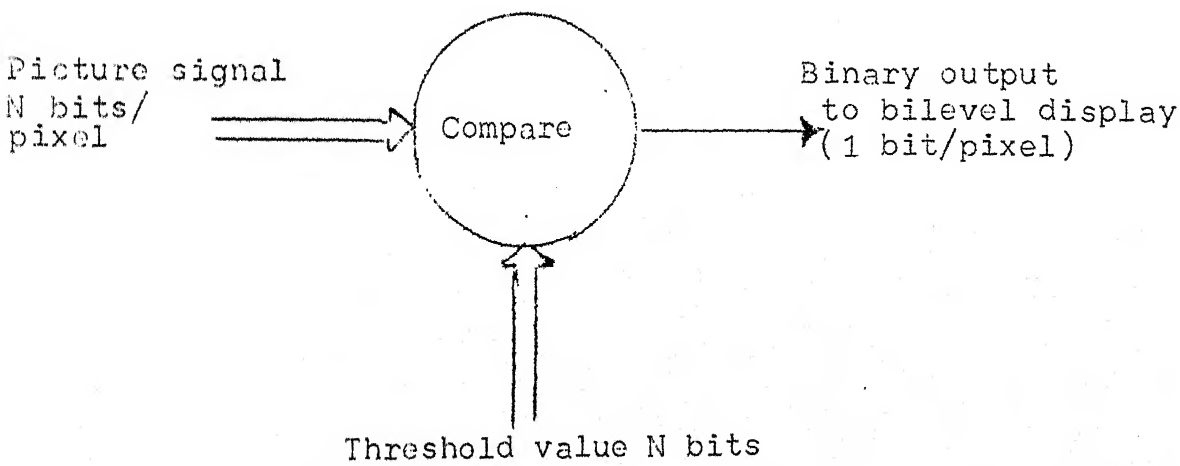

Fig. 3.2 The basic processing organisation

scales may also become visible. The 4x4 patterns used for the generation of the picture in Fig. 2a of Appendix A are visible. In certain cases, depending upon the selection of fonts,texture contours might be visible in the output picture. There is no processing complexity as the technique simply requires a pixel look up table.

The set of patterns suggested by Hamill were modified into 6x6 binary patterns to represent 32 gray levels. The picture generated by this set is as shown in Fig. 2b of Appendix A. Using the same set of 6x6 patterns another picture was generated and is shown at Fig. 9 of Appendix A. The quality of this picture is poorer than the picture of Fig. 2b of Appendix A. This was caused due to a programming error. A similar error in the program using Hamill's 4x4 patterns resulted in the absolutely distorted picture of Fig. 8 of Appendix A. In this, the data has been compressed and repeated to give the distortion where smaller sized pictures of Lincoln have been superimposed on the original picture.

3.3 TECHNIQUES WITH EQUAL INPUT AND OUTPUT RESOLUTION

The basic processing organisation that produces the binary output for the generation of the output picture is shown in Fig. 3.2. Intensity of each pixel of the input picture is compared with a threshold signal. If the intensity of the pixel is greater than the threshold the

corresponding output pixel is set to bright state, otherwise the output pixel is set to dark state. The various techniques described in this section differ primarily in the way they produce the threshold values for comparison with the intensity of the input pixel.

3.3.1 Fixed Threshold Technique

Fig. 3.3 shows a schematic diagram for a globally fixed level thresholding process with input I(x,y) and output O(x,y). This technique is simplest to implement in that the decision rule is simply stated as

$$\text{If} \quad I(x,y) > T; \quad \text{then} \quad O(x,y) = 1$$
$$\text{else} \quad O(x,y) = 0$$

The threshold T is predecided and globally fixed and is generally in the neighbourhood of the centre value in the tone scale (16 for the test picture of Lincoln). The output process simply generates black and white pixels, depending upon the gray level of the input image at that location. Fig. 3a of Appendix A shows a picture generated by this method. As is obvious, the two ends of the gray scale are reproduced accurately but the remainder are severely distorted. The output picture either has dark or bright areas.

Fine detail which has gray level swings above and below the threshold will be seen. Hence, mid range fine

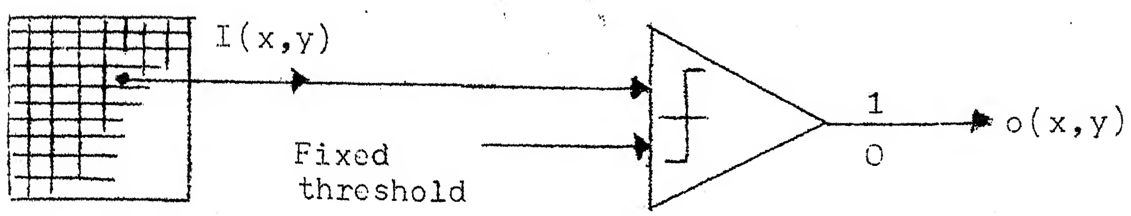

Fig. 3.3 Signal flow diagram for globally fixed threshold

(a) $$\begin{bmatrix} 0 & 14 & 3 & 13 \\ 11 & 5 & 8 & 6 \\ 2 & 12 & 1 & 15 \\ 9 & 7 & 10 & 4 \end{bmatrix}$$

$$\begin{array}{cccc|cccc} 0 & 16 & 4 & 20 & 1 & 17 & 5 & 21 \\ 24 & 8 & 28 & 12 & 25 & 9 & 29 & 13 \\ 6 & 22 & 2 & 18 & 7 & 23 & 3 & 19 \\ 30 & 14 & 26 & 10 & 31 & 15 & 27 & 11 \\ \hline 1 & 17 & 5 & 21 & 0 & 16 & 4 & 20 \\ 25 & 9 & 29 & 13 & 24 & 8 & 28 & 12 \\ 7 & 23 & 3 & 19 & 6 & 22 & 2 & 18 \\ 31 & 15 & 27 & 11 & 30 & 14 & 26 & 10 \end{array}$$

Fig. 3.4 Dither matrices (a) proposed by Lippel and Kurland (b) proposed by Bayer

details is reproduced but the details away from the mid-range are never reproduced. Though the detail rendition is limited to a very small range, very high frequency details are enabled in that range. A high contrast text can be displayed effectively by this technique but not a multi-gray level picture. The processing complexity of this technique is minimum and only one pixel of 'context' is required for the computation.

Fig. 3b of Appendix A shows a picture that has been generated in a slightly different manner. Instead of comparing a pixel of input picture, an average of the pixel and its eight immediate neighbours is calculated and compared to the fixed threshold. As seen, the quality of the output picture still remains the same. The processing complexity of this technique is more as nine pixels of 'context' are required to compute the window average.

3.3.2 Ordered Dither Technique

This technique was worked out for TV signals which being analog had to be quantized before digital transmission and due to the need to reduce the number of bits transmitted, the obvious solution was to reduce the number of quantization steps by increasing the step size. This resulted in the degradation of the TV picture in the areas where intensity changed slowly (i.e. low detail areas). The degradation took the form of 'false contours'. To eliminate

these contours it was suggested to add a pseudo noise signal called a dither signal to the true input signal. This produced rapid switching between the quantizer levels on either side of the true input signal. Pictures were transmitted by using as few as 4 bits/pixel (i.e., 16 gray levels) by Roberts [2] by adding a 2 dimensional pseudo noise sequence. The false contouring in the received picture was reduced by the fact that the subject viewed the picture as a near continuum of gray levels by the eye's averaging over a small two dimensional area.

In the present context the multilevel picture is being quantized into just two levels and one may add a two dimensional pseudo noise sequence to an input prior to quantization to two levels. The resulting picture has the errors distributed spatially and the observer integrates the average reflectance in a small region and sees a near continuum of gray levels. Limb [3], Lippel and Kurland [4] have used it for binary representation. It has been shown subjectively that 'ordered dither patterns' constructed with a small square dither matrix repeated over the picture area to form a rectangular array of matrices is superior to a random arrangement of the same population of samples. Pictures generated by the two kinds of dither have been studied by Lippel and Kurland [4]. They have also studied the arrangement of dither samples in the dither matrix to

find out optimum dither patterns keeping in mind the effect the pattern has on pictorial information in the output picture. The dither patterns suggested by them are given in Fig. 3.4(a). Bayer has also suggested a dither pattern given in Fig. 3.4(b).

This technique thus generates a bilevel representation of continuous tone images by comparing the image values I(x,y) to a position dependant set of thresholds contained in a square dither matrix. This dither matrix is repeated over the entire picture in a checkerboard fashion and the decision for a pixel to be turned bright or dark is

$$\text{If } I(x,y) > D(x,y), \text{ then } O(x,y) = 1$$
$$\text{else } O(x,y) = 0$$

Jarvis et al [5] have shown a recursive relationship between the 2x2 matrix of Limb and the matrix generated by Bayer. The Limb's matrix D^2 is one of the four 2x2 matrices that satisfy the Bayer optimization criteria.

$$\text{Given } D^2 = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} \tag{3.1}$$

and Defining

$$U^n = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \tag{3.2}$$

The recursive relationship is :

$$D^u = \begin{bmatrix} 4D^{n/2} + D^2_{oo}\ U^{n/2} & 4D^{n/2} + D^2_{o1}\ U^{n/2} \\ 4D^{n/2} + D^2_{10}\ U^{n/2} & 4D^{n/2} + D^2_{11}\ U^{n/2} \end{bmatrix} \qquad (3.3)$$

By the first recursion, we get

$$D^4 = \begin{bmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{bmatrix} \qquad (3.4)$$

This matrix scaled by a factor of 2 to cover the entire range of 32 levels has been used to generate the picture of Fig. 4a and Fig. 4c of Appendix A. Fig. 4a is a 128x128 picture whereas Fig. 4c is a 256x256 picture. Bayer's matrix of Fig. 3.4b has been used to generate the picture of Fig. 4b of Appendix A. The pictures of Figs.4d, 4e and 4f of Appendix A have been generated using the matrix of (3.4) above but without any scaling to cover the entire range of the tone scale.

This technique results in complete gray level scale generation in the output picture. Comparing pictures of Figs. 4a and 4c of Appendix A with the picture generated by Bayer's matrix (Fig. 4b), it is seen that Bayer's **matrix** produces more detail. It is due to the entire gray scale range being covered in steps of 1 as compared to the

steps of two for the pictures of Fig. 4a and 4c. The pictures of Figs. 4d, 4e and 4f miss out half the tonal scale due to error in the matrix used. However, picture of 4f gives slightly more detail due to the randomness of the data used.

The processing complexity of this technique is slightly more than the fixed threshold technique as here also only one pixel is processed at a time and instead of a globally fixed threshold it has a look up table for the thresholds.

3.3.3 Constrained Average Technique

This technique developed by Jarvis and Roberts [6] generates one bit for each pixel of input picture $I(x,y)$ by comparing its intensity to a threshold value T computed from $I(x,y)$ and its eight nearest neighbours in the array. of the input pixels. If $I(x,y)$ is greater than the threshold T, the corresponding pixel at the output is turned bright. This technique provides edge emphasis alongwith gray scale rendition. The process uses the finite signal to noise ratio inherent in the picture data due to sampling and quantization errors, to generate, on a statistical basis a number of bright cells related to the average image intensity in the picture area.

The threshold is computed as :

$$T = \gamma + \bar{I}(x,y)\left[1 - \frac{2\gamma}{R}\right] \tag{3.5}$$

where $\bar{I}(x,y)$ is the average intensity of $I(x,y)$ and its eight nearest neighbours, R is the highest gray level (31 in the case of the test picture of Lincoln) and γ is a parameter which controls the apparent contrast of the output image and is related to noise statistics. Since $\bar{I}(x,y)$ varies from 0 to R, the threshold changes in the value from γ ro $R-\gamma$.

The working of the technique can be explained by the following arguments. When the average intensity in an area is equal to R/2, the threshold value from (3.5) is also equal to R/2 and an approximately equal number of pixels will have values above and below the threshold T resulting in one half of the pixels being turned bright. As the brightness increases, the threshold T is less than the average brightness $\bar{I}(x,y)$ resulting in more than one-half of the pixels being turned bright. The reverse happens when the brightness decreases.

The techniques capability of enhancing edges is seen when a pixel markedly different from the local average intensity is considered. The pixel will be turned on or off depending upon its intensity value relative to the neighbourhood average. Pixels near a dark line (edge) will have higher probability of being turned on as the neighbourhood average will be reduced due to the presence of the dark pixels belonging to the line. The reverse will hold good for the pixels on the dark line itself as then the

neighbourhood average would be more than the value of the pixel and it will be turned off. This suggests that the size of the area considered for the computation of the neighbourhood average and the threshold should be small as any step intensity changes or gradients in the area effect the state of the output pixels. Roberts [6] has shown that the constant γ depends upon the noise characteristics of the signal. Thus, a usable value of γ can be obtained if we know the noise characteristics of the signal being processed. In practice, γ may be thought of as an arbitrary parameter that can be specified to obtain a pleasing rendition of the original picture. $\gamma > 0$ results in normal gray scale rendition.

Fig. 3.5 shows a signal flow diagram for the constrained average threshold technique. Figs. 5a to 5e and 6a and 6b of Appendix A show the various pictures generated with different types of data, and different values of γ. As can be seen, edge emphasis has been achieved in all the pictures irrespective of the value of γ or the type of data (repetitive or random) used. For $\gamma = 0$, the picture generated only emphasizes the edges whereas pictures with $\gamma > 0$ render edge emphasis alongwith the gray scales. The control over the apparent contrast of the technique is visible in pictures 5b and 5c. Picture 5c has more contrast than 5b. With $\gamma < 0$, the generated picture is a negative

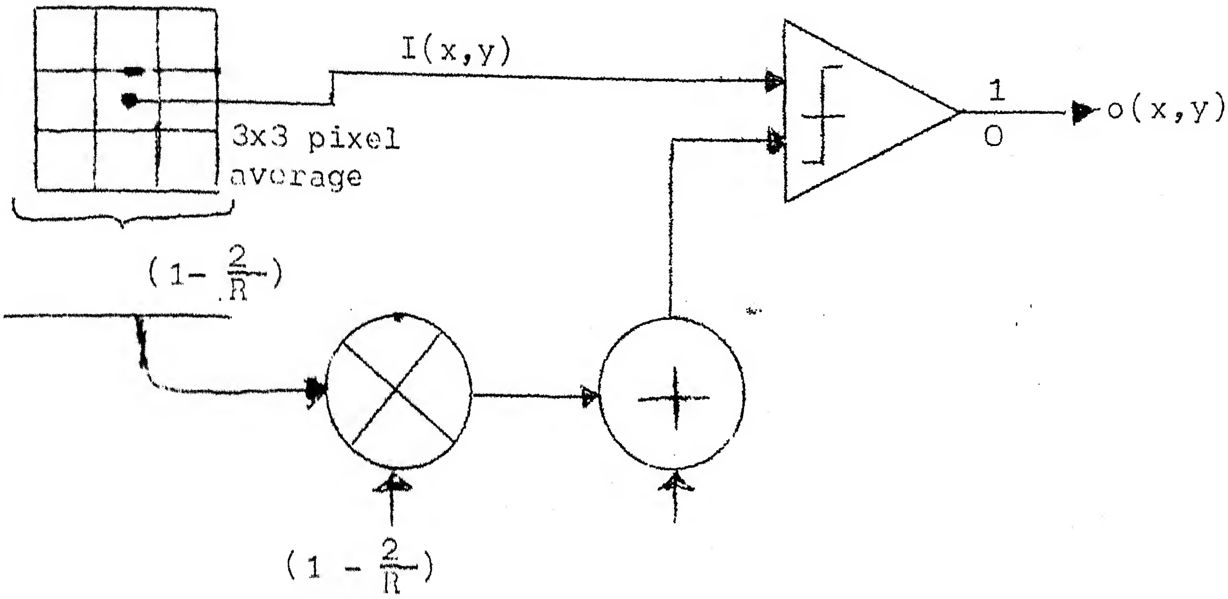

Fig. 3.5 Signal processing flow diagram for constrained Average technique

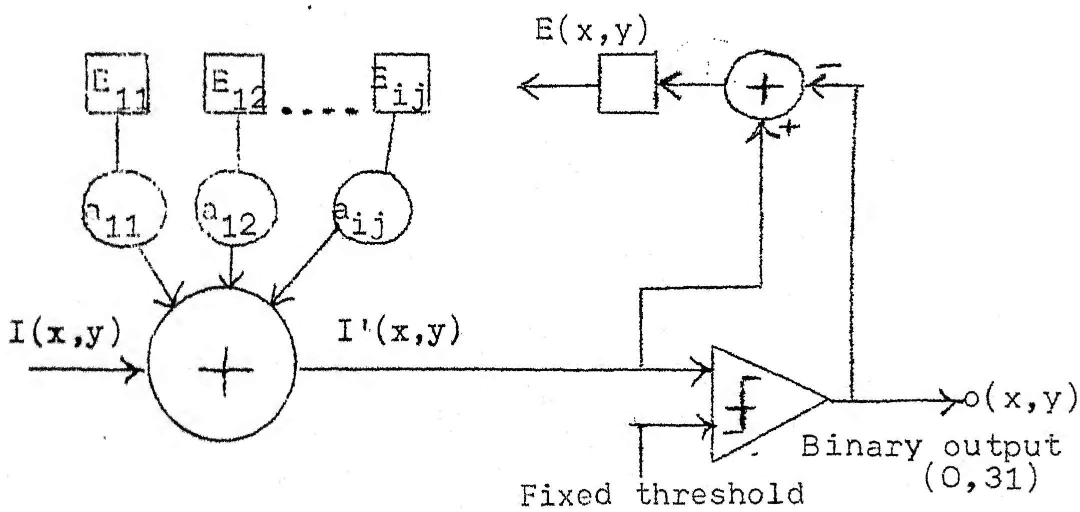

Fig. 3.6a Signal flow diagram for the error diffusion method

$$a_{ij} = \begin{vmatrix} 3 & 6 & 10 & 6 & 3 \\ 6 & 10 & 15 & 10 & 6 \\ 10 & 15 & * & - & - \end{vmatrix}$$

Fig. 3.6(b) The size of the error matrix

like picture but it is not a true negative as the edge enhancement still turns on cells that are more positive than their neighbourhood average. Finally, picture of 5f has been generated by adding a dither signal having rectangular distribution to the data of the picture. This gives the best effects of gray scales and maximum details. The complexity of the constrained Average Technique is more than both the fixed threshold and ordered dither techniques, as a 'context' of 9 pixels is required to compute the neighbourhood average and the technique requires further processing to calculate the threshold.

3.3.4 Error Diffusion Technique

This technique generates local patterns of binary pixels in such a way as to minimize the intensity error between the source and displayed image. When a pixel which was initially gray is turned to bright or dark, it constitutes an error in gray scale. Thus to correct the picture, this gray error is redistributed to nearby pixels, restoring the total gray content to its original value. The original work was presented by Schroeder [7] where the pictures were processed with the help of more than two output states. The technique was used for binary displays by Floyd [8]. Given the gray level of input pixel, the binary decision about the output pixel is made and the error between these two levels is dispersed (diffused) to the

right and below the processed pixel. Fig. 3.6a shows a signal flow diagram representation of the error diffusion technique.

At a point (x,y) in the picture there will be an error between the displayed intensity and the original image which contains more intensity levels than the displayed image.

$$E(x,y) = I(x,y) - O(x,y)$$

where $E(x,y)$ is the intensity error at a point (x,y), $O(x,y)$ is the displayed intensity and $I(x,y)$ is the original image intensity. To implement the algorithm, a corrected intensity $I'(x,y)$ is computed from the previously computed errors and the intensity $I(x,y)$:

$$I(x,y) = I(x,y) + \sum_{ij} a_{ij} E(i,j)$$

In this equation, the ranges of the indices i and j are defined by the definition of the neighbourhood of point $I(x,y)$. a_{ij} define the weights that multiply the gray scale errors $E(i,j)$ for those pixels prior to the addition of gray scale for $I(x,y)$. Fig. 3.6b gives the neighbourhood of the point (x,y) and the weights a_{ij} that multiply the various $E(i,j)$. The corrected intensity is then compared to a fixed threshold (centre value of the tone scale) to determine whether the corresponding display element should be bright or dark. That is,

If $I'(x,y) > T$: Then $O(x,y) = 1$
else $O(x,y) = 0$.

Finally the error is computed as

$$E(x,y) = I'(x,y) - O(x,y).$$

The element marked * in Fig. 3.6b is the position of the point $I(x,y)$ under consideration and does not enter into the error computation. Likewise, the elements marked - are ahead of the point (x,y) in the scan line and no error for these elements exists; hence, they can not enter into the error computation. Jarvis et al [5] have shown empirically that the minimum size of the matrix (neighbourhood) needed to generate subjectively satisfactory results is of the order shown in Fig. 3.6b. Hence, the context of this technique is the order of (ij). In the present case, 'context' is 12, as twelve error values are needed to compute the binary output. The complexity of the technique can also be visualised from the fact that it requires 3 line memories to contain the successive error values and also a substantial number of computations for each pixel are needed, as a weighted sum is required to be done. The boundary handling consists of setting the error values to zero for a boundary two pixels deep on each side and the top of the picture. Only the pixels inside this region are evaluated. This two pixel width boundary is normally not

visible in a 128x128 or higher picture.

This technique, like the ordered dither technique, is capable of reproducing the complete tone scale but the spatial frequencies in the output image will be a function of both the gray level being reproduced and the input image detail. This is due to the effect of the error diffusion to the neighbourhood. For example, the spatial distance between black pixels in a bright area will be very long as compared to that distance for mid-tone gray levels (due to the difference in the quantum of error in the two cases). There may also be a spatial translation in the location of an edge detail depending upon the image content above and to the left of the edge.

This technique is capable of creating output signals which have lower spatial frequencies (as mentioned above) and are thus more visible than in the case of ordered dither technique, which puts a limit on the low frequency response.

Figs. 7a to 7c of Appendix A are the pictures generated by this technique. Picture of 7a is a 64x64 picture whereas the remaining pictures are 128x128. The picture of Fig. 7c has been generated from the random data and hence has more noise as compared to the picture of Fig. 7b which has been generated from the repetitive data.

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CHAPTER IV

CONCLUSION

Electronic manipulation of the pictorial imagery for storage, communication and reproduction often use binary marking/display processes for cost effectiveness. The binary data generated by these processes may be further coded to affect more saving. To illustrate, the picture of Lincoln after being converted to binary bits is Run Length Coded (RLC) and again reproduced. The program used to carry out this processing is named RLC. FOR and is attached at Appendix B. The picture of Lincoln generated by this program is attached as Fig. 10 of Appendix A. The picture is an exact replica of the constrained average picture of Fig. 5b of Appendix A which was taken as the binary picture for coding. The saving effected by the constrained average process was to reduce the data to 1/5th of its original size and the run length coding further affected a saving of slightly less than half.

Though the possibility of displaying pictures with full range of gray scales as a series of on or off cells on a grid of moderate resolution appears marginal, the pictures reproduced in this study show that usable pictures can be displayed in this fashion and hence provide a range

of options to the designer of a visual communication system using bilevel displays.

The processed pictures generated by the various algorithms have been explained in each of the sections/ sub-sections under which the algorithms have been discussed. The techniques are evaluated as under :

4.1 EVALUATION OF TECHNIQUES

(a) Orthographic Technique :

The frequency rendition is restricted by the factors of m and n as the binary patterns used are of the size mxn. It is quite common to find false contouring caused due to the binary patterns used. It can be seen to a little extent in the picture of Fig. 2 of Appendix A.It will be more prominent when a higher gray level picture is reproduced by this technique. The output picture also contains a texture depending upon the patterns used. The binary pattern used can easily be seen in the picture of Fig. 2 of Appendix A.

(b) Fixed Threshold Technique :

The frequency rendition of this technique is restricted to only the mid-range gray levels. However, very high frequency fine details are rendered in this range. The black/white boundary in the output picture is an ever present and most common artefact (false contour) to be seen.

Low noise riding the gray levels near the threshold value gets amplified. This low noise may, in fact, be due to the inherent noise characteristics (practical limits on signal to noise factor) of the scanning systems. It may result into a visible artefact which was not present in the original picture.

(c) Ordered Dither :

Due to the checkerboard kind of arrangement of the dither matrix throughout the picture, a limit is placed on the low frequency content of the output. The technique, but for this limit, produces a complete range of frequency and tone scale. The most critical artefact that is seen in this technique is the arteficial texture that often accompanies the image. This texture is visible in the dark areas of the coat and bow of Lincoln. To develop an algorithm which gives both high detail rendition and minimum textural visibility is a topic of ongoing research.

(d) Constrained Average Technique :

High frequencies (fine details) are obtained by this algorithms due to its inherent capability of edge enhancement. This detail, though, is at the cost of edge noise. Edges appear more blurred than in the case of fixed threshold due to the use of a neighbourhood average.

(e) Error Diffusion Technique :

This technique produces the complete tonal scale. The spatial frequencies are a function of both the gray level being reproduced and the input image detail. There may be a spatial translation in the location of edge detail depending upon the image content above and to the left of the edge. The technique produces lower frequencies which are more visible than in the case of the ordered dither technique. This effect is clearly visible in the pictures of Figs. 7b and 7c.

The control of the spatial frequencies of the intensity errors by the ordered dither technique provides the best rendition of the complete tonal scale in the original image. This is seen clearly when the dither outputs are compared to the outputs of the other techniques.

Second conclusion reached is that beside the fixed level threshold, the constrained Average Technique due to its inherent edge enhancing capability will make the detail in image consisting of lines and text more visible.

Jarvis et al [1] have made a comparison of the various algorithms and presented it in the form of a picture showing the comparison of the gray scale capabilities of the various techniques. They have reached the conclusion that Ordered Dither besides being a simple technique to implement, also gives the best results, and, perhaps constrained average with dither is a strong contestant to being the best.

Appx A.

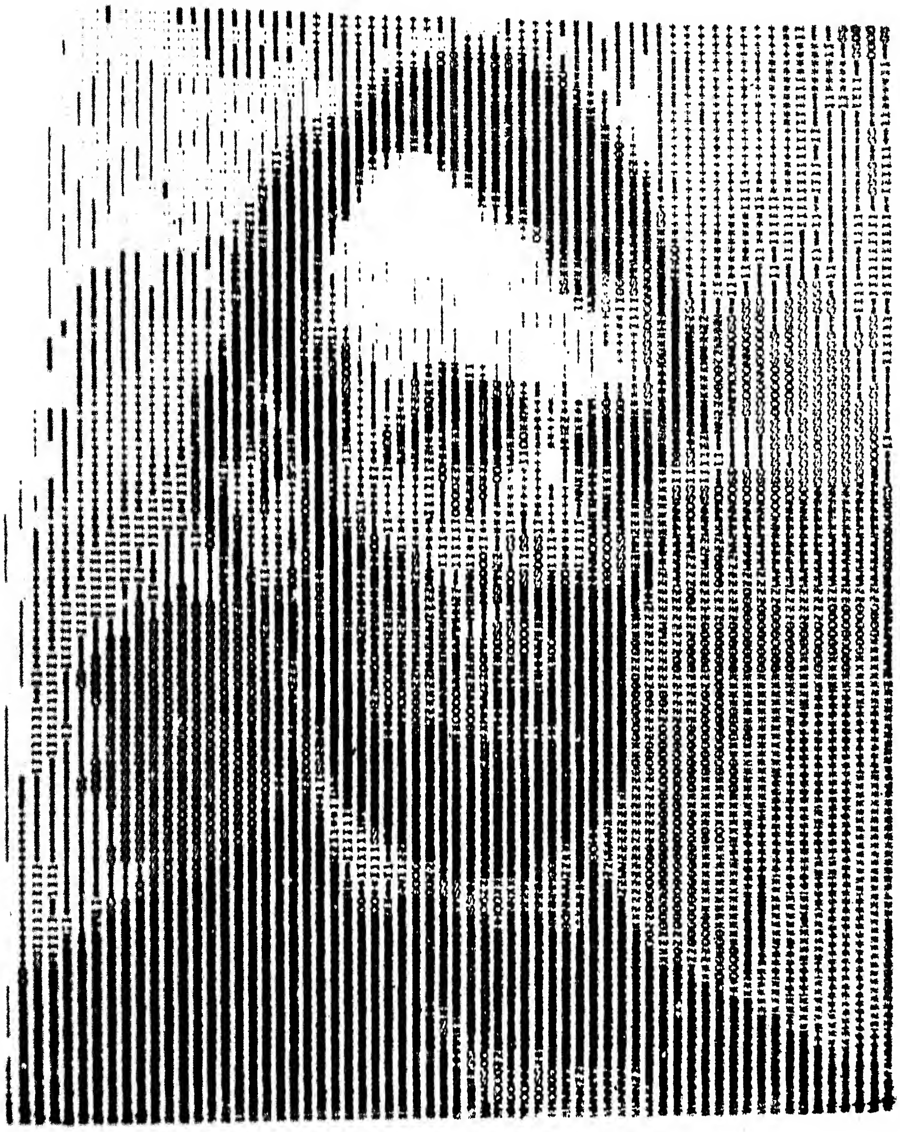
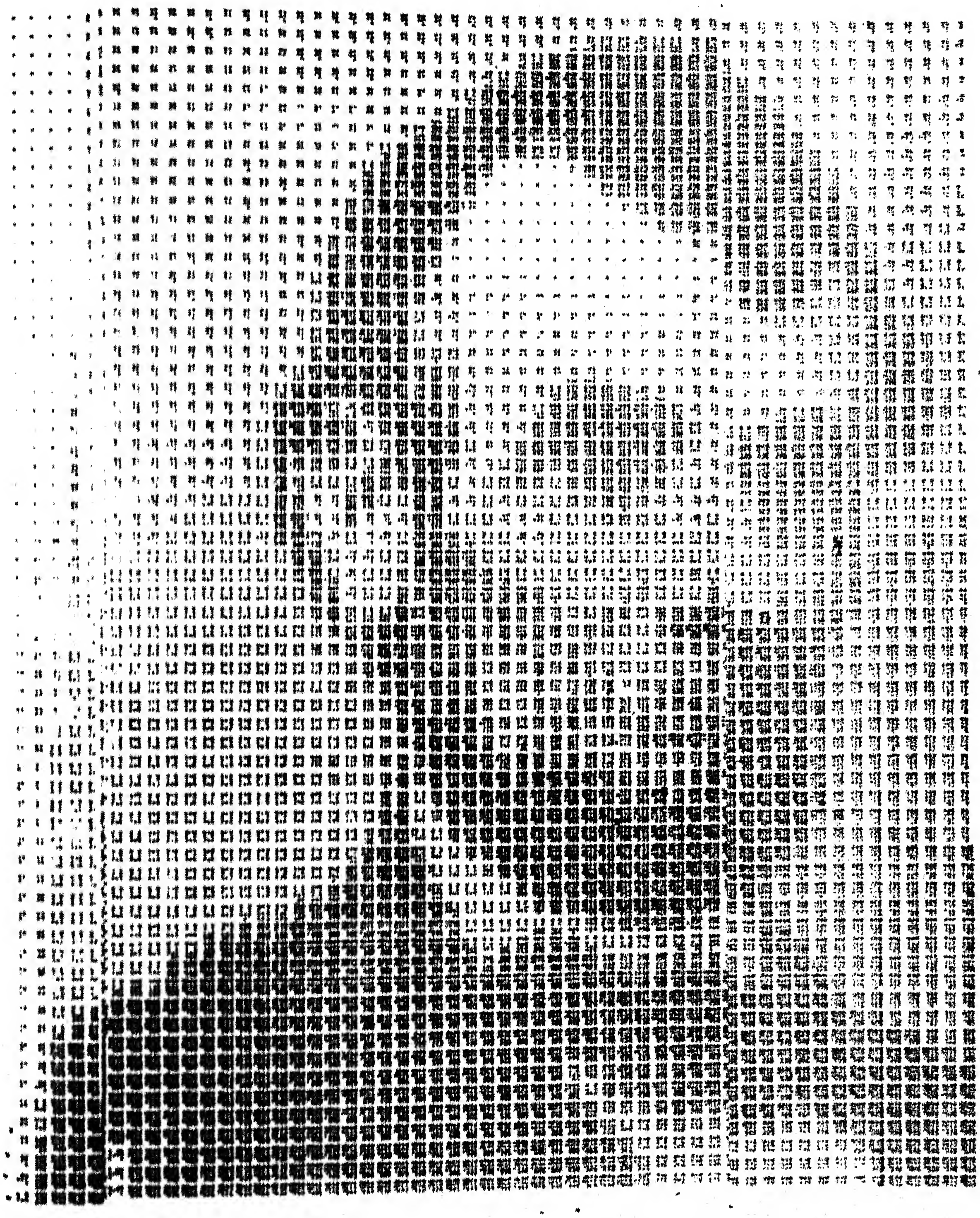
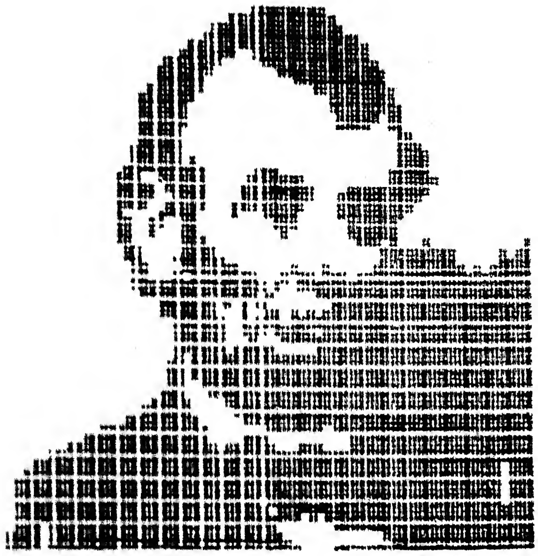


FIG 1: The Test Picture of Lincoln (64x64).

FIG 2: Orthographic Technique: using 4x4 Binary Patterns

SPSS-10





(a)

(b)

FIG 3: Fixed Threshold Technique.

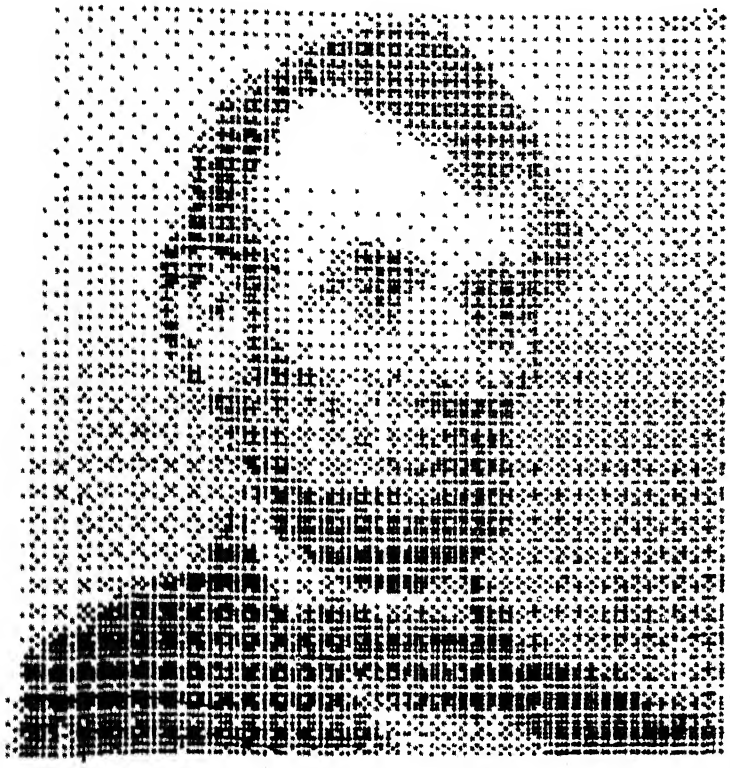


FIG 4(a): Ordered Dither Using D4

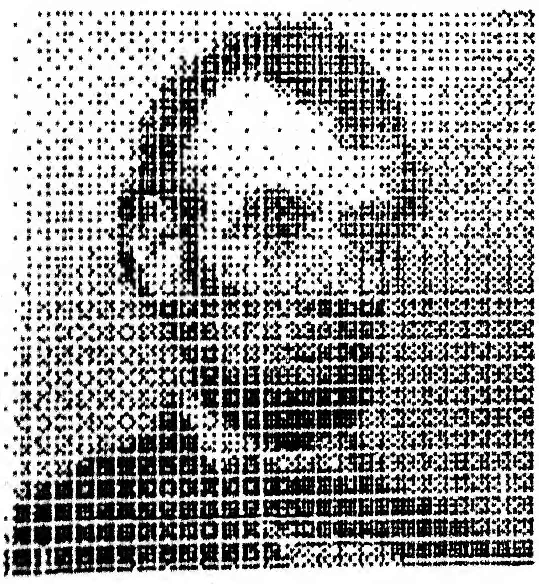


FIG 4(b): Ordered Dither Using Bayer's Matrix.
(128 x 128)

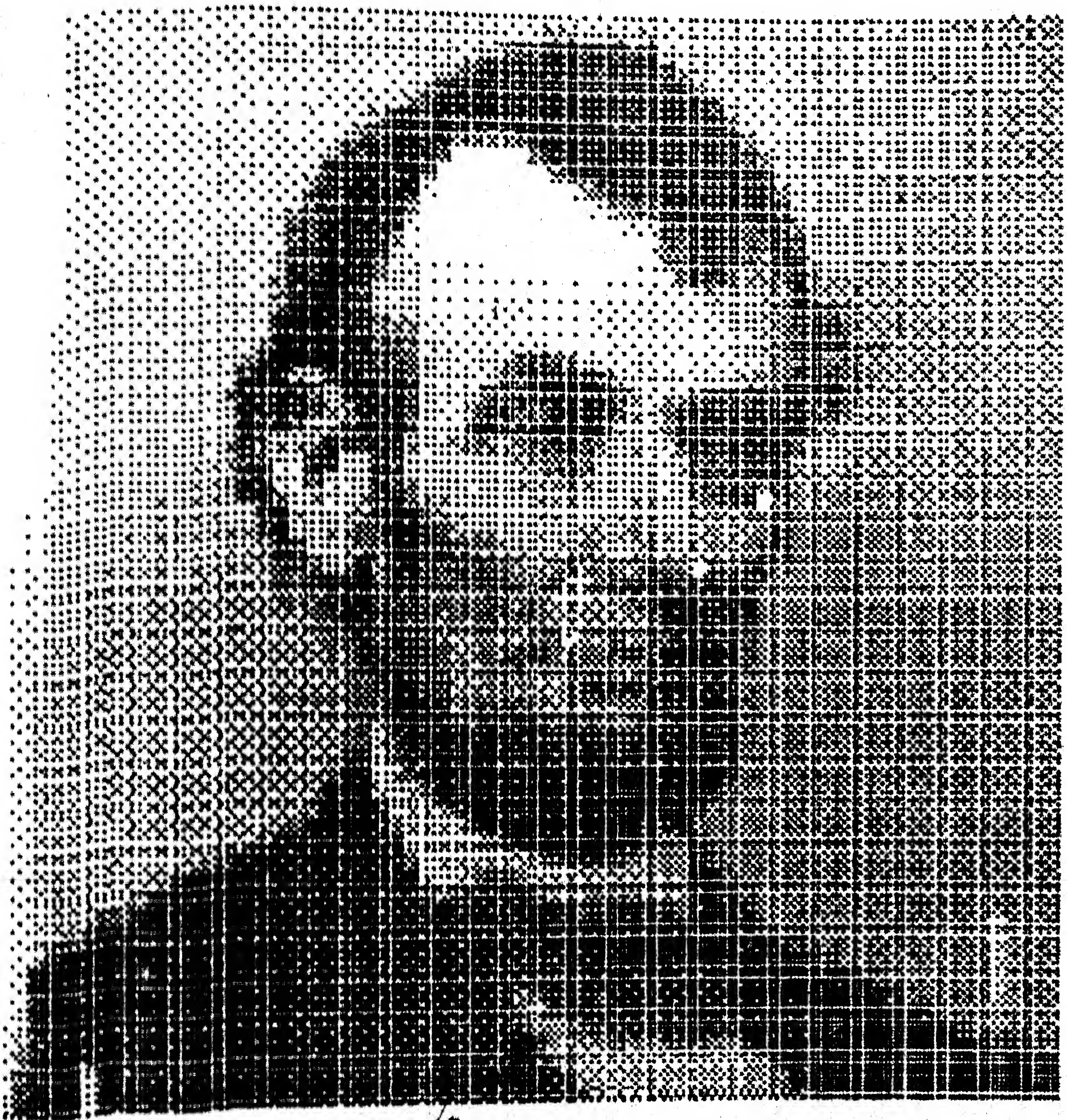


Fig 4(C) - 256×256 Repetitive Data.

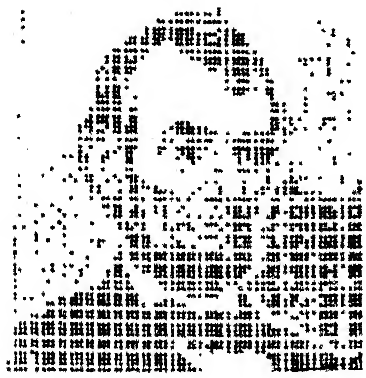
FIG 4(d): Ordered Dither Using D^4 With Half the Range (256 x 256)

(e)

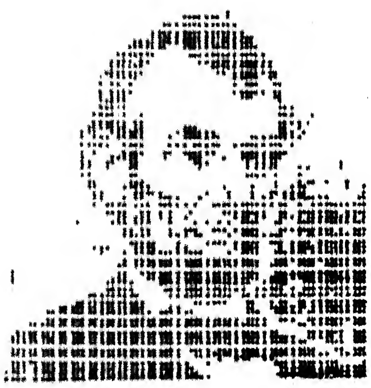
(f)

Fig 4 (contd.) -. (e) Repetitive Data (f) Random Data.

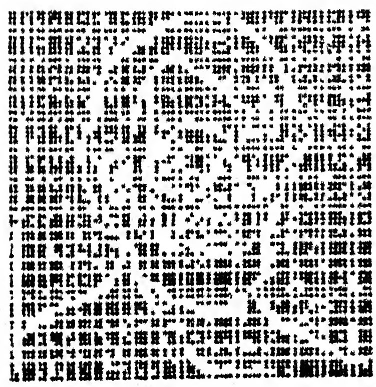
(a) $\gamma = 2$; X·GE·Y



(b) $\gamma = 2$; X·GT·Y

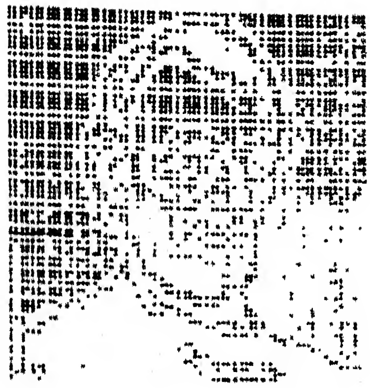


(c) $\gamma = 4$; X·GT·Y

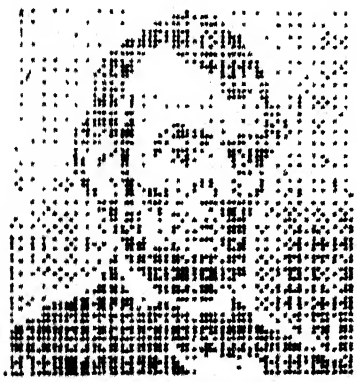


(d) $\gamma = 0$; X·GT·Y

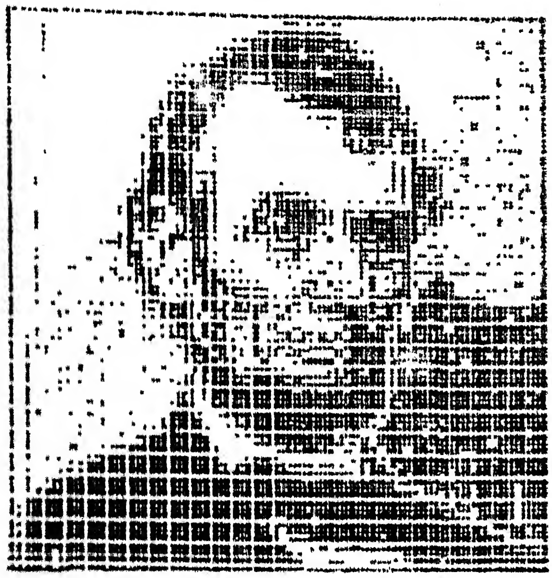
FIG 5: Constrained Average Technique
(64 x 64)



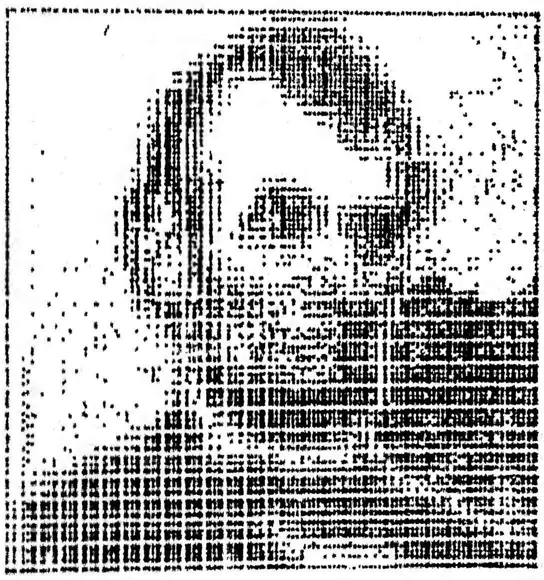
(e) $\nu = -2$; X.GT.Y



; (contd.) - (f) With Added Dither.

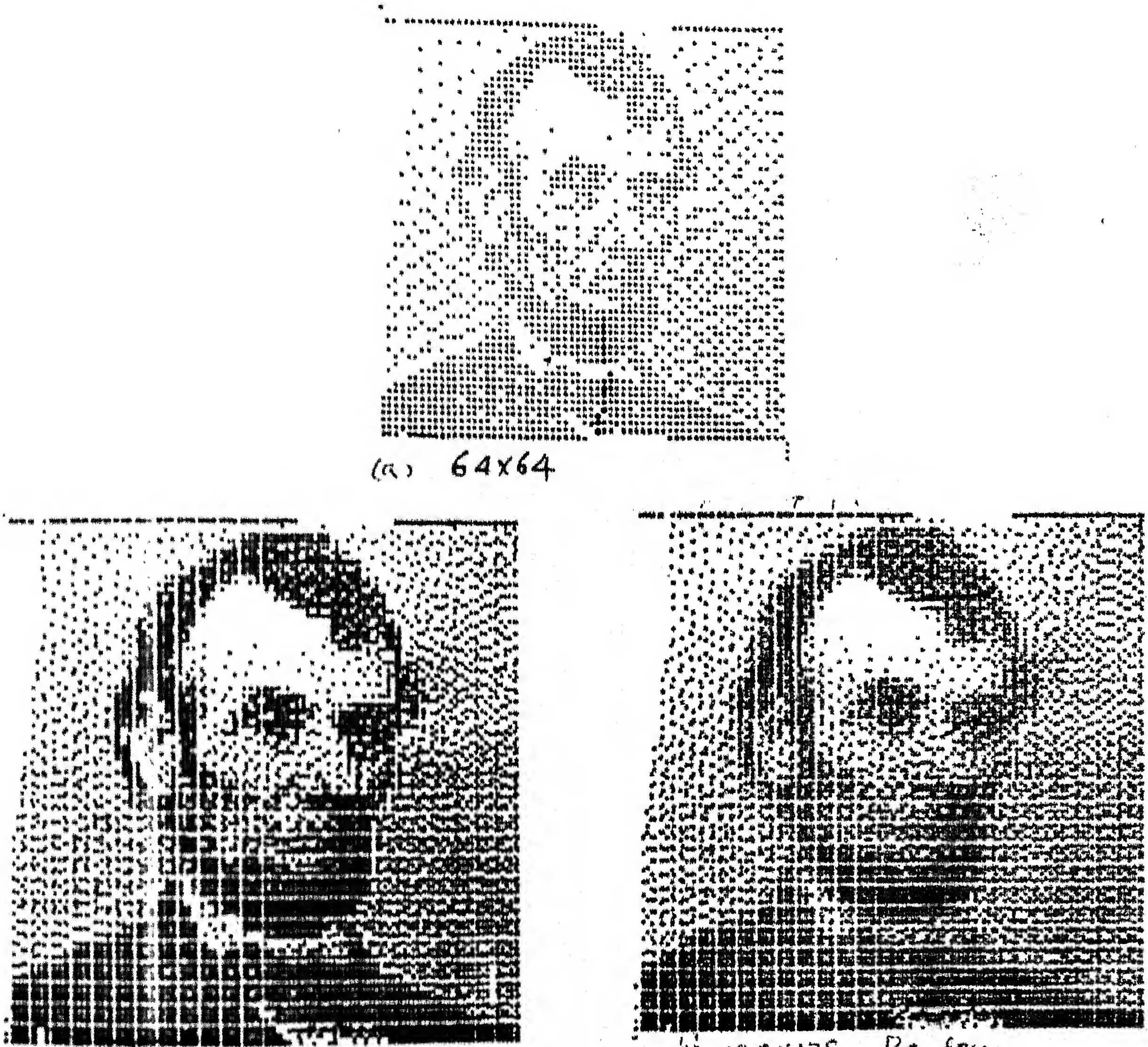


(a) Repeated Data (128x128)



(b) Random Data (128x128)

FIG 6: Constrained Average



(a) 64x64

(b) 128x128 Repetitive.

(c) 128x128 Random.

FIG 7: Error Diffusion Technique.

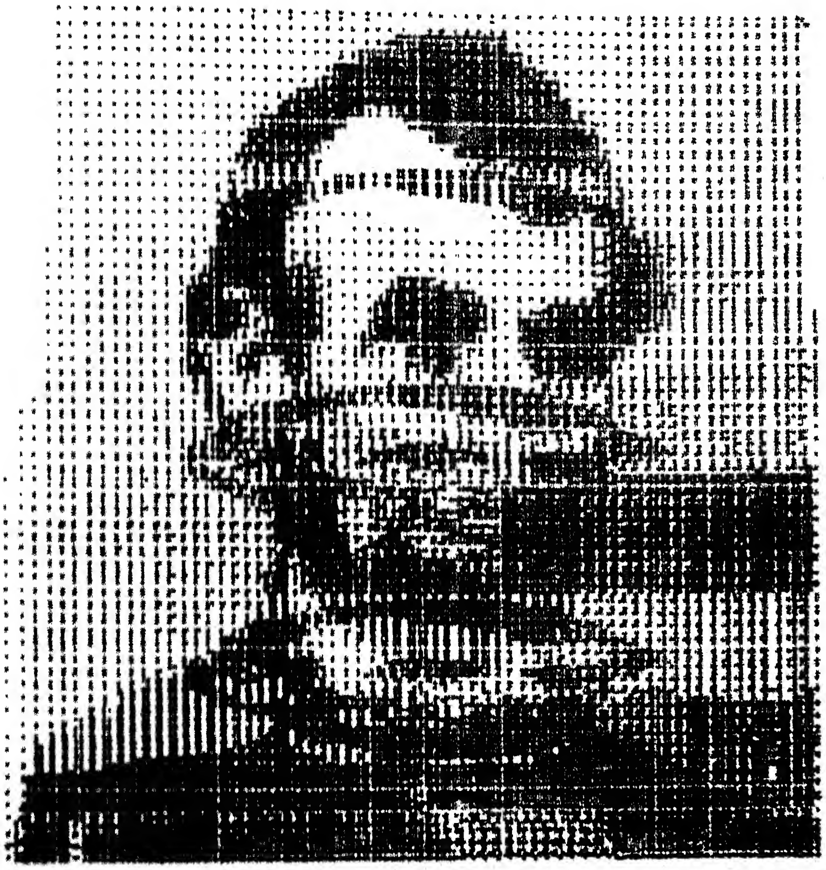


FIG 8: Orthographic Technique

FIG 9: Orthographic Technique Using 6x6 Patterns

Appx B.

```
C           PROGRAM DIST.FOR
C           ----------------
C           GAUSS DISTRIBUTION
C           NAG G05DDF
            INTEGER A(64,64)
            OPEN(UNIT=22,DEVICE=DSKD,FILE='FOR22.DAT')
            OPEN(UNIT=23,DEVICE=DSKD,FILE='FOR23.DAT')
            READ (22,*),((A(I,J),J=1,64),I=1,64)
            AVERAG=0;DEVIAT=0;
            IT=1
            DO 50JT=1,5
            DO 51IREP=1,16
             WRITE(23,20),A(IT,JT)
    51      CONTINUE
    50      CONTINUE
            DO 200I=2,4
            WRITE(23,20),(A(I,1),K=1,16)
            DO 300J=2,4
            AVERAG=FLOAT(A(I-1,J-1)+A(I-1,J)+A(I-1,J+1)+A(I,J-1)+A(I,J)
           1+A(I,J+1)+A(I+1,J-1)+A(I+1,J)+A(I+1,J+1))/9.+0.5
            DEVIAT=ABS(AVERAG-FLOAT(A(I,J)))
            CALL G05CBF(0)
            ICOUNT=0
    10      X=G05DDF(AVERAG,DEVIAT)
            IAVERA=AVERAG
            INTX=X
            IF((INTX.GE.A(I,J)).AND.(INTX.GE.IAVERA))GO TO 450
    20      FORMAT(10X,I5)
            GO TO 10
    450     WRITE(23,20),INTX
            ICOUNT=ICOUNT+1
            IF(ICOUNT.EQ.16)GO TO 300
            GO TO 10
    300     CONTINUE
            WRITE(23,20),(A(I,10),K=1,16)
    200     CONTINUE
            IB=10
            DO 52JB=1,5
            DO 53IREP=1,16
            WRITE(23,20),A(IB,JB)
    53      CONTINUE
    52      CONTINUE
            STOP
            END
```

```
      PROGRAM FDTST.FOR
      -----------------
      DIMENSION A(64,16),B(255)
      INTEGER A,B
      OPEN(UNIT=24,DEVICE='DSK0',FILE='FOR24.DAT')
      DO 5L=1,64
      DO 10I=1,64
      READ(23,*),(A(I,J),J=1,16)
      DO 11J=1,4
      B(4*(I-1)+J)=A(I,J)
11    CONTINUE
10    CONTINUE
      WRITE(24,*),(B(I),I=1,255)
      DO 20I=1,64
      DO 21J=5,8
      B(4*(I-1)+J-4)=A(I,J)
21    CONTINUE
20    CONTINUE
      WRITE(24,*),(B(I),I=1,255)
      DO 30I=1,64
      DO 31J=9,12
      B(4*(I-1)+J-8)=A(I,J)
31    CONTINUE
30    CONTINUE
      WRITE(24,*),(B(I),I=1,256)
      DO 40I=1,64
      DO 41J=13,16
      B(4*(I-1)+J-12)=A(I,J)
41    CONTINUE
40    CONTINUE
      WRITE(24,*),(B(I),I=1,256)
5     CONTINUE
15    FORMAT(256I3)
      STOP
      END
```

```
C     PROGRAM CONVOL.FOR
C     -----------------
C     THIS PROGRAM GENERATES THE RESPONSE OF THE
C     EYE MODEL.THE PROGRAM  PERFORMS THE TWO
C     DIMENSIONAL CONVOLUTION WITH THE INPUT
C     3*3 BINARY PATTERNS.
C     THE SYSTEM RESPONSE HAS BEEN  RESTRICTED
C     TO 21*21 MATRIX ONLY.THE CONVOLVED OUTPUT
C     IS A 23*23 MATRIX.
      DIMENSION AX(21,21),A(3,3),O(23)
      CK(X,Y)=1.25312*EXP(-0.7*(X+Y))-0.05125*EXP(-0.0714*(X+Y))
      N=10;READ *,((A(I,J),J=1,3),I=1,3)
      DO 1 I=1,3
      DO 1 J=1,3
      A(I,J)=ABS(A(I,J))
1     CONTINUE
      PRINT 2
2     FORMAT(5X,'THE INPUT PATTERN IS',/5X,'
     1--------------------')
      PRINT*,((A(I,J),J=1,3),I=1,3)
      N1=N+1
      AX(N+1,N+1)=CK(0,0)
      X=0.
      DO 5 J=1,N
      X=X+0.02
      C1=CK(X,0.)
      AX(N+1,N+1+J)=C1;AX(N+1,N+1-J)=C1
      AX(N+1+J,N+1)=C1;AX(N+1-J,N+1)=C1
5     CONTINUE
      Y=0.
      DO 7 I=1,N
      Y=Y+0.02;X=0.
      DO 7 J=1,N
      X=X+0.02;C1=CK(X,Y);NI=N1+I;NJ=N1+J;JN=N1-J;IN=N1-I
      AX(NI,NJ)=C1;AX(NI,JN)=C1
      AX(IN,NJ)=C1;AX(IN,JN)=C1
7     CONTINUE
      N2=2*N+1
      PRINT 3
3     FORMAT(10X,'THE SYSTEM RESPONSE MATRIX IS',
     1/10X,'-------------------------')
      PRINT 13,((AX(I,J),J=1,N2),I=1,N2)
13    FORMAT(5X,21F6.2)
      PRINT 4
4     FORMAT(10X,'THE SYSTEM RESPONSE IS',/10X,
     1'------------------------')
      DO 10 I=1,N2+2
      DO 9 J=1,N2+2
      SUM=0.
      DO 8 K=1,3
      DO 8 L=1,3
      I1=I+1-K
      J1=J+1-L
      IF(I1.LE.0.OR.J1.LE.0)GO TO 8
      IF(I1.GT.21.OR.J1.GT.21)GO TO 8
      SUM=SUM+AX(I1,J1)*A(K,L)
8     CONTINUE
      O(J)=SUM
9     CONTINUE
      PRINT 14,(O(J1),J1=1,N2+2)
14    FORMAT(5X,23F5.2)
10    CONTINUE
      STOP;END
```

```
      DIMENSION X(25),Y(25),XP(25),YP(25),TEMP(25),W(6),V(6),Z(25)
      INTEGER TTY
      DATA TTY/7/,W/-2.0,6.0,0.0,8.0,-2.0,6.0/
      DATA V/0.0,1.0,0.0,1.0,0.0,1.0/
      CALL NITDEV(TTY)
      CALL IDEN
      CALL WINDW(W(1))
      CALL VPORT(V(1))
C     CALL CLICTL(1)
      DO 5L=1,18
      READ(20,*),((X(I),Y(I),Z(I)),I=1,5)
      N=5
      SLP1=30.
      SLPN=330.
      Z1=Z(1)
      CALL ROTAD(20.0,2)
      CALL ROTAD(20.0,1)
      CALL LINE3(X(1),Y(1),Z1,0)
      WRITE(24,*),X(1),Y(1),Z1
      CALL IDEN
      S=0.;SIGMA=0.1
      CALL KURV1(N,X,Y,SLP1,SLPN,XP,YP,TEMP,S,SIGMA)
      T11=0.01;XS=0;YS=0;
      DO 100IJ=1,100
      IF(IJ.GT.1)T11=-T11
      T=T11*FLOAT(IJ)
      CALL KURV2(T,XS,YS,N,X,Y,XP,YP,S,SIGMA)
      IF(IJ-(IJ/2)*2.NE.0)GO TO 100
      CALL ROTAD(20.0,2)
      CALL ROTAD(20.0,1)
      CALL LINE3(XS,YS,Z1,1)
      WRITE(24,*),XS,YS,Z1
      CALL IDEN
100   CONTINUE

5     CONTINUE
      DO 6L=1,22
      READ(21,*),((X(I),Y(I),Z(I)),I=1,5)
      N=5
      SLP1=30.
      SLPN=330.
      X1=Z(3)
      CALL ROTAD(20.0,2)
      CALL ROTAD(20.0,1)
      CALL LINE3(X1,Y(1),X(1),0)
      WRITE(24,*),X1,Y(1),X(1)
      CALL IDEN
      S=0.;SIGMA=0.1
      CALL KURV1(N,X,Y,SLP1,SLPN,XP,YP,TEMP,S,SIGMA)
      T11=0.01;XS=0;YS=0;
      DO 101I=1,100
      IF(I.GT.1)T11=-T11
      T=T11*FLOAT(I)
      CALL KURV2(T,XS,YS,N,X,Y,XP,YP,S,SIGMA)
      IF(I-(I/2)*2.NE.0)GO TO 101
      CALL ROTAD(20.0,2)
      CALL ROTAD(20.0,1)
      CALL LINE3(X1,YS,XS,1)
      WRITE(24,*),X1,YS,XS
      CALL IDEN
101   CONTINUE
6     CONTINUE
      ACCEPT*,OK
      STOP
      END
C     THIS SUBROUTINE DETERMINES THE PARAMETERS NECESSARY TO
C     COMPUTE A SPLINE UNDER TENSION PASSING THROUGH A SEQUENCE
C     OF PAIRS (X(1),Y(1),---------,X(N),Y(N))IN THE PLANE.THE
C     SLOPES AT THE TWO ENDS OF THE CURVE MAY BE SPECIFIED OR
C     OMITTED.FOR ACTUAL COMPUTATION OF POINTS ON THE CURVE IT
C     IS NECESSARY TO CALL THE SUBROUTINE KURV2.
C     ON INPUT --------
C     N IS THE NUMBER OF POINTS TO BE INTERPOLATED (N.GE.2)
C     X IS AN ARRAY CONTAINING THE N X-COORDINATES OF THE
C     POINTS
C     Y IS AN ARRAY CONTAINING THE N Y-COORDINATES OF THE
C     POINTS
C     SLP1 AND SLPN CONTAIN THE DESIRED VALUES FOR THE SLOPES
C     OF THE CURVE AT (X(1),Y(1)) AND (X(N),Y(N)),RESPECTIVELY.
C     THESE QUANTITIES ARE IN DEGREES AND MEASURED COUNTER-
C     CLOCKWISE FROM THE POSITIVE X-AXIS.THE POSITIVE SENSE
C     OF THE CURVE IS ASSUMED TO BE THAT MOVING FROM THE
C     POINT 1 TO POINT N.IF THE QUANTITY SIGMA IS NEGATIVE
C     THESE SLOPES WILL BE DETERMINED INTERNALLY AND THE USER
C     NEED ONLY FURNISH PLACE-HOLDING PARAMETERS FOR SLP1
C     AND SLPN.SUCH PLACE HOLDING PARAMETERS WILL BE IGNORED
C     BUT NOT DESTROYED.
C     YP AND XP ARE ARRAYS OF LENGTH AT LEAST N,
```

```
C         SEARCH STORAGE,
C         AND
C         SIGMA CONTAINS THE TENSION FACTOR.THIS IS NON ZERO AND
C         INDICATES THE CURVINESS DESIRED.IF ABS(SIGMA)IS VERY
C         LARGE(E.G.50) THE RESULTING CURVE IS VERY NEARLY A
C         POLYGONAL LINE.THE SIGN OF SIGMA INDICATES WHETHER
C         SLOPE INFORMATION HAS BEEN INPUT OR NOT.IF SIGMA IS
C         NEGATIVE THE END POINT SLOPES WILL BE DETERMINED
C         INTERNALLY.A STANDARD VALUE FOR SIGMA IS APPROXIMATELY
C         1.IN ABSOLUTE VALUE.
C         ON OUTPUT------
C         N,X,Y,SLP1,SLPN AND SIGMA ARE UNALTERED,
C         XP AND YP CONTAIN INFORMATION ABOUT THE CURVATURE OF THE
C         CURVE AT GIVEN NODES,
C         AND
C         S CONTAINS THE POLYGONAL ARC LENGHT OF THE CURVE.
          SUBROUTINE KURV1(N,X,Y,SLP1,SLPN,XP,YP,TEMP,S,SIGMA)
          INTEGER N
          REAL X(N),Y(N),XP(N),YP(N),TEMP(N),S,SIGMA
          DEGRAD =3 .1459265/180.
          NM1=N-1
          NP1=N+1
          DELX1=X(2)-X(1)
          DELY1=Y(2)-Y(1)
          DELS1=SQRT(DELX1*DELX1+DELY1*DELY1)
          DX1=DELX1/DELS1

          DY1=DELY1/DELS1
C         DETERMINE SLOPES IF NECESSARY
          IF (SIGMA.LT.0.) GO TO 70
          SLPP1=SLP1*DEGRAD
          SLPPN=SLPN*DEGRAD
C         SET UP RIGHT HAND SIDES OF TRIDIAGONAL LINEAR SYSTEM
C         FOR XP AND YP
10        XP(1)=DX1-COS(SLPP1)
          YP(1)=DY1-SIN(SLPP1)
          TEMP(1)=DELS1
          S=DELS1
          IF(N.EQ.2) GO TO 30
          DO 20 I=2,NM1
          DELX2=X(I+1)-X(I)
          DELY2=Y(I+1)-Y(I)
          DELS2=SQRT(DELX2*DELX2+DELY2*DELY2)
          DX2=DELX2/DELS2

          DY2=DELY2/DELS2

          XP(I)=DX2-DX1
          YP(I)=DY2-DY1
          TEMP(I)=DELS2
          DELX1=DELX2
          DELY1=DELY2
          DELS1=DELS2
          DX1=DX2
          DY1=DY2
C         ACCUMULATE POLYGONAL ARCLENGHT
          S=S+DELS1
20        CONTINUE
30        XP(N)=COS(SLPPN)-DX1
          YP(N)=SIN(SLPPN)-DY1
C         DENORMALISE TENSION FACTOR
          SIGMAP=ABS(SIGMA)*FLOAT(N-1)/S
C         PERFORM FORWARD ELIMINATION ON TRIDIAGINAL SYSTEM
          DELS=SIGMAP*TEMP(1)
          EXPS=EXP(DELS)
          SINHS=0.5*(EXPS-1./EXPS)
          SINHIN=1./(TEMP(1)*SINHS)
          DIAG1=SINHIN*(DELS*0.5*(EXPS+1./EXPS)-SINHS)
          DIAGIN=1./DIAG1
          XP(1)=DIAGIN*XP(1)
          YP(1)=DIAGIN*YP(1)
          SPDIAG=SINHIN*(SINHS-DELS)
          TEMP(1)=DIAGIN*SPDIAG
          IF(N.EQ.2) GO TO 50
          DO 40 I=2,NM1
          DELS=SIGMAP*TEMP(I)
          EXPS=EXP(DELS)
          SINHS=0.5*(EXPS-1./EXPS)
          SINHIN=1./(TEMP(I)*SINHS)
          DIAG2=SINHIN*(DELS*(0.5*(EXPS+1./EXPS))-SINHS)
          DIAGIN=1./(DIAG1+DIAG2-SPDIAG*TEMP(I-1))
          XP(I)=DIAGIN*(XP(I)-SPDIAG*XP(I-1))
          YP(I)=DIAGIN*(YP(I)-SPDIAG*YP(I-1))
          SPDIAG=SINHIN*(SINHS-DELS)
          TEMP(I)=DIAGIN*SPDIAG
          DIAG1=DIAG2
40        CONTINUE
50        DIAGIN=1./(DIAG1-SPDIAG*TEMP(NM1))
```

```
      YP(N)=DIAGIN*(YP(N)-SPDIAG*YP(NM1))
C     PERFORM BACK SUBSTITUTION
      DO 60 I=2,N
      IBAK=NP1-I
      XP(IBAK)=XP(IBAK)-TEMP(IBAK)*XP(IBAK+1)
      YP(IBAK)=YP(IBAK)-TEMP(IBAK)*YP(IBAK+1)
   60 CONTINUE
      RETURN
   70 IF(N.EQ.2) GO TO 80
C     IF NO SLOPES ARE GIVEN,USE SECOND ORDER INTERPOLATION ON
C     INPUT DATA FOR SLOPES AT END POINTS
      DELS2=SQRT((X(3)-X(2))**2+(Y(3)-Y(2))**2)
      DELS12=DELS1+DELS2
      C1=-(DELS12+DELS1)/DELS12/DELS1
      C2=DELS12/DELS1/DELS2
      C3=-DELS1/DELS12/DELS2
      SX=C1*X(1)+C2*X(2)+C3*X(3)
      SY=C1*Y(1)+C2*Y(2)+C3*Y(3)
      SLPP1=ATAN2(SY,SX)
      DELNM1=SQRT((X(N-2)-X(NM1))**2+(Y(N-2)-Y(NM1))**2)
      DELN=SQRT((X(NM1)-X(N))**2+(Y(NM1)-Y(N))**2)
      DELNN=DELNM1+DELN
      C1=(DELNN+DELN)/DELNN/DELN
      C2=-DELNN/DELN/DELNM1
      C3=DELN/DELNN/DELNM1
      SX=C3*X(N-2)+C2*X(NM1)+C1*X(N)
      SY=C3*Y(N-2)+C2*Y(NM1)+C1*Y(N)
      SLPPN=ATAN2(SY,SX)
      GO TO 10
C     IF ONLY TWO POINTS AND NO SLOPES ARE GIVEN,USE STRAIGHT
C     LINE SEGMENT FOR CURVE
   80 XP(1)=0.
      XP(2)=0.
      YP(1)=0.
      YP(2)=0.
      RETURN
      END
C     THIS SUBROUTINE PERFORMS THE MAPPING OF POINTS IN THE
C     INTERVAL OF(0.,1.) ONTO A CURVE IN THE PLANE.THE SUB-
C     ROUTINE KURV1 SHOULD BE CALLED EARLIER TO DETERMINE
C     CERTAIN NECESSARY PARAMETERS.THE RESULTING CURVE HAS
C     A PARAMETRIC REPRESENTATION BOTH OF WHOSE COMPONENTS
C     ARE SPLINE UNDER TENSION AND FUNCTIONS OF THE POLYGON-
C     AL ARCLENGHT PARAMETER.
C     ON INPUT ------
C     T CONTAINS A REAL VALUE OF ABSOLUTE VALUE LESS THAN OR
C     EQUAL TO 1.TO BE MAPPED TO A POINT ON THE CURVE.THE
C     SIGN OF T IS IGNORED AND THE INTERVAL (0.,1.)IS MAPPED
C     ONTO THE ENTIRE CURVE.IF T IS NEGATIVE THIS INDICATES
C     THAT THE SUBROUTINE HAS BEEN CALLED PREVIOUSLY(WITH ALL
C     OTHER INPUT VARIABLES UNALTERED) AND THAT THIS VALUE OF
C     T EXCEEDS THE PREVIOUS VALUE IN ABSOLUTE VALUE.WITH
C     SUCH INFORMATION THE SUBROUTINE IS ABLE TO MAP THE POINT
C     MUCH MORE RAPIDLY.THUS IF THE USER SEEKS TO MAP A
C     SEQUENCE OF POINTS ON TO THE SAME CURVE,EFFICIENCY IS
C     GAINED BY ORDERING THE VALUES INCREASING IN MAGNITUDE
C     AND SETTING THE SIGN OF ALL BUT THE FIRST NEGATIVE.
C     N CONTAINS THE NUMBER OF POINTS WHICH WERE INTERPOLATED
C     TO DETERMINE THE CURVE.
C     X AND Y ARE ARRAYS CONTAINING THE X-AND Y-COORDINATES
C     OF THE INTERPOLATED POINTS.
C     XP AND YP ARE THE ARRAYS OUTPUT FROM KURV2 CONTAING
C     CURVATURE INFORMATION.
C     S CONTAINS THE POLYGONAL ARCLENGHT OF THE CURVE.
C     SIGMA CONTAINS THE TENSION FACTOR (ITS SIGN IS IGNORED).
C     THE PARAMETERS N,X,Y,XP,YP,S AND SIGMA SHOULD BE INPUT
C     UNALTERED FROM THE OUTPUT OF KURV1.
C     ON OUTPUT -----
C     XS AND YS CONTAIN THE X- AND Y-COORDINATES OF THE IMAGE
C     POINT ON THE CURVE.
C     T,N,X,Y,XP,YP,SAND SIGMA ARE UNALTERED
C     DENORMALIZE SIGMA
      SUBROUTINE KURV2(T,XS,YS,N,X,Y,XP,YP,S,SIGMA)
      INTEGER N
      REAL T,XS,YS,X(N),Y(N),XP(N),YP(N),S,SIGMA
      SIGMAP=ABS(SIGMA)*FLOAT(N-1)/S
C     STRETCH UNIT INTERVAL INTO ARC LENGHT DISTANCE
      TN=ABS(T*S)
C     FOR NEGATIVE T START SEARCH WHERE PREVIOUSLY TERMINATED
C     OTHERWISE START FROM BEGINNING
      IF(T.LT.0.) GO TO 10
      I1=2
      XS=X(1)
      YS=Y(1)
      SUM=0.
      IF(S.LE.0.) RETURN
```

```
      C         DETERMINE INTO WHICH SEGMENT TN IS MAPPED
                DO 30 I=I1,N
                DELX=X(I)-X(I-1)
                DELY=Y(I)-Y(I-1)
                DELS=SQRT(DELX*DELX+DELY*DELY)
                IF(SUM+DELS-TN) 20,40,40
      20        SUM=SUM+DELS
      30        CONTINUE
      C         IF ABS(T) IS GREATER THAN 1.,RETURN TERMINAL
      C         POINT ON CURVE
                XS=X(N)
                        YS=Y(N)
                RETURN
      C         SET UP AND PERFORM INTERPOLATION
      40        DEL1=TN-SUM
                DEL2=DELS-DEL1
                EXPS1=EXP(SIGMAP*DEL1)
                SINHD1=0.5*(EXPS1-1./EXPS1)
                EXPS=EXP(SIGMAP*DEL2)
                SINHD2=0.5*(EXPS-1./EXPS)
                EXPS=EXPS1*EXPS
                SINHS=0.5*(EXPS-1./EXPS)
                XS=(XP(I)*SINHD1+XP(I-1)*SINHD2)/SINHS
                1+((X(I)-XP(I))*DEL1+(X(I-1)-XP(I-1))*DEL2)/DELS
                YS=(YP(I)*SINHD1+YP(I-1)*SINHD2)/SINHS
                1     +((Y(I)-YP(I))*DEL1+(Y(I-1)-YP(I-1))*DEL2)/DELS
                I1=I
                RETURN
                END
      C         THIS SUBROUTINE PERFORMS THE MATRIX MULTIPLICATION
      C         OF A AND B MATRICES TO GIVE C MATRIX AS THE RESULT
                SUBROUTINE MATMUL (A,B,C)
                DIMENSION A(4,4),B(4,4),C(4,4)
                DO 10 I=1,4
                DO 10 J=1,4
                C(I,J)=0.0
                DO 10 K=1,4
                C(I,J)=C(I,J)+A(I,K)*B(K,J)
      10        CONTINUE
                RETURN
                END
                SUBROUTINE MATPRI(P)
                DIMENSION P(4,4)
                WRITE(23,10),((P(I,J),J=1,4),I=1,4)

      10        FORMAT(4F16.10)
                RETURN
                END
```

```
C      This program generates  a crosshatched figure,
C      that is, one figure whose lines run parallel to the X-axis
C      overlayed by another figure whose lines run parallel to the
C      Z-axis.
C
       DIMENSION MASK(2000), VERTEX(20), OUTBUF(51), Z(51),Y(51)
       DIMENSION W(4),X1(50,51),Y1(50,51),Z1(50,51)
       DATA W/0.0,1000.0,0.0,1000.0/
       CALL NITDEV(7)
       CALL WINDOW(W)
C      First figure
C      Generate data running parallel to X-axis
       DO 1 I=1,29
       DO 1 J=1,51
       READ(22,*),X1(I,J),Y1(I,J),Z1(I,J)
1      CONTINUE

         DO 20 NLINE = 1, 29
           DO 10 NPOINT = 1, 51
       X(NPOINT)=X1(30-NLINE,NPOINT)
       OUTBUF(NPOINT)=Y1(30-NLINE,NPOINT)
       Z(NPOINT)=Z1(30-NLINE,NPOINT)
10           CONTINUE
C      Plot each line as it is computed
       CALL PLOT3D(111,X, OUTBUF,Z,0.2, 0.2, 0.2,
       1NLINE,51,-20.0,-20.0, 0.0,5.0, 10.0, MASK, 0)
20     CONTINUE
C      Second figure
C      Generate data running parallel to Z-axis
        DO 50 NLINE=1,29
          DO 40 NPOINT=1,51
       READ(23,*),X(52-NPOINT),OUTBUF(52-NPOINT),Z(52-NPOINT)
40        CONTINUE
C      Plot each line as it is computed
         CALL PLOT3D( 1111, X, OUTBUF, Z, 0.2, 0.2,0.2,
       1   NLINE, 51,-20.0,-20.0,0.0,5.0, 10.0, MASK, VERTEX )
50       CONTINUE
C      Draw a frame on the figure.
C        CALL FRAMER( 3, VERTEX, MASK )
       ACCEPT*,OK
         STOP; END
C
C
```

```
      SUBROUTINE PLOT3D( IVXYZ, XDATA, YDATA, ZDATA, XSCALE,
     1  YSCALE, ZSCALE, NLINE, NPNTS, PHI, THETA, XREF,
     2  YREF, XLENTH, MASK, VERTEX )
C     Masked 3-Dimensional plot program with rotations
C     This routine will accepts 3-Dimensional data in various
C     forms as input, rotate in 3-space to any angle
C     and plot the projection of the resulting figure onto the
C     XY-plane. Linear interpolation is used between data points.
C     Those lines of a figure which should be hidden by a previous
C     line are masked.
C     The masking techniques used by this routine is based on
C     two premises -
C           Lines in the fore-ground ( positive Z-direction )
C           are plotted before lines in the background.
C           A line or a portion of a line is masked ( hidden ) if
C           it lies within the region bounded by previously
C           plotted lines.
C     Each call to PLOT3D causes one line of a figure to be plotted.
C      Two parameters of the plotter are set on the initial call
C     for each figure -
C     ( PIPI ) is the number of plotter increments per inch.
C     ( NYPI ) is the number of increments available accross the
C     width of the paper ( Y-direction ).
C     When a new figure is initiated, the plotter origin is set
C     at the bottom of the paper by PLOT3D and should not be
C     moved until the figure is completed.
C     Input parameters -
C     ( IVXYZ ) is a four digit decimal integer which is used
C     to select various input/output options. These digits in
C     decreasing order of magnitude, will be referred to as V,
C     X, Y, and Z.
C     If V.NE.0, the verticies of the current figure and their
C     projection on to the Y = 0 plane, will be stored in a 16
C     entry real array ( VERTEX ) and will be updated as each
C     line is plotted. These co-ordinates are in inches and
C     relative to the current plotter origin. The X-Y pairs
C     are ordered so that first pair corresponds to the
C     first point of the figure, the second pair corresponds to
C     the last point of the first line, and the following
C     pairs are ordered in a circular fashion. The pairs on the
C     Y = 0 plane of the figure, then follow in the same order.
C     If V = 0, the VERTEX parameter is ignored, but should not be
C     deleted.
C     If X = 0, the X-components of this line are assumed to be
C     equally spaced, and are computed by
C           X( I ) = XDATA(1) + ( I-1 )*XSCALE
C     where ( XDATA ) is the initial value in inches and ( XSCALE )
C     is the spacing between points in inches. If X.NE.0, the
C     X-components of this line are read from an array and
C     modified by
C           X( I ) = XDATA(I)*XSCALE
C     where XSCALE is a scale factor.
C     The same relations hold for the Y-components, i.e., if
C     Y = 0
C           Y( I ) = YDATA(1) + (I-1)*YSCALE
C     and if Y.NE.0
C           Y( I ) = YDATA(I)*YSCALE
C     If Z = 0, the Z-components of this line are all assumed to
C     be equal, and are computed by
C           Z( I ) = ZDATA(1) + (NLINE-1)*ZSCALE
C     where ( NLINE ) is some integer associated with this line.
C     If Z.NE.0, again we have
C           Z( I ) = ZDATA(I)*ZSCALE
C     When ( NLINE ) is equal to one, it indicates the beginning
C     of a new figure. A call to PLOT3D with ( NLINE ) equal to
C     zero before initiating a new figure simulates a line drawn
C     at the bottom of the page. Therefore only those portions
C     of a line lying above all previous lines will be plotted.
C     all other parameters are ignored on such a call.
C     ( NPNTS )  is the number of points on this line and may be
C     altered from line to line.
C     ( PHI ) and ( THETA ) are the two angles ( in degrees ) used to
C     specify the desired 3-dimensional rotations. The following
C     two definitions of these rotations are equivalent -
C     In terms of rotations of axes, the initial system of axes,
C     XYZ, is rotated by an angle ( PHI ) counter-clockwise about
C     the Y-axis, and the resultant system is labelled to TUV
C     axes. The TUV axes are then rotated by an angle ( THETA )
C     counter-clockwise about the T-axis, and this final system
C     is labelled the PQR-axes. The plotted figure is the
C     projection of the original figure onto the PQ-plane.
C     In terms of rotations of co-ordinates, the figure is first
C     rotated by an angle ( THETA ) clockwise about the X-axis.
C     The resultant figure is then rotated by an angle ( PHI )
C     clockwise about its Y-axis. The plotted figure is the
C     projection of this final figure on to the XY-plane.
C     Warning:
C       Some rotations will alter the fore-ground/background
C       relationships between the lines, and thus the order
```

```
C       in which they should be plotted.
C       ( XREF ) and ( YREF ) are the co-ordinates in inches,
C       relative to the plotter origin, to be used as the origin
C       of the figure.
C       ( XLENTH ) is the length, in inches, to which the plot is
C       restricted. Any points which exceeds this limit, or the
C       limits of the paper in the Y-direction ( NYPI ) will be
C       set to that limit.
C       ( MASK ) is an integer array of 2*XLENTH*PIPI entries which
C       is used to store the MASK. The contents of this array
C       should not be altered during the plotting of any given
C       figure.
C       All parameters except ( MASK ) and ( VERTEX ) are returned
C       unchanged.
C       Between any two calls for the same figure, any parameter
C       can be meaningfully changed except ( XLENTH ), ( MASK ), and
C       ( VERTEX ).
        INTEGER HIGH, OLDHI, OLDLOW
        DIMENSION XDATA( 1 ), YDATA( 1 ), ZDATA( 1 ), MASK( 1 ),
       1VERTEX(1)
        DATA INIT, JVXYZ, SPHI, STHETA/ -1, -1, -1.0E35, -1.0E35/
C       Initialization procedures
C       Initialization procedure for a new figure
C       Test for special MASK modifying call
          IF( NLINE.EQ.0 ) GOTO 550
C       Determine if initialization is required
          IF( NLINE.NE.1 ) GOTO 20
C       Set plotter parameters
          PIPI = 100.0
          NYPI = 1090
C       Reset plotter origin to bottom of plot page
          I = NYPI+100
          CALL IPLOT( 0, 0, 3 )
C       Compute length of plot page in increments
          LIMITX = XLENTH*PIPI + 0.5
          I = LIMITX + LIMITX
C       Initialize masking array over the length of the plot page
          DO 10 K=1,I
            MASK(K) = INIT
10        CONTINUE
          INIT = -1
C       Set the necessary indicators for the first line of a new
C       figure
          INCI = -1; I = 0
C       Input type and VERTEX initialization
C       Determine if initialization is required
20        IF( JVXYZ.EQ.IVXYZ ) GOTO 70
C       Set indicators for types of input data and saving verticies
          JVXYZ = IVXYZ
          INDZ = 1; INDY = 1; INDX = 1; INDV = 1
          IF( JVXYZ.LT.1000 ) GOTO 30
          INDV = 2
          JVXYZ = JVXYZ - 1000
30        IF( JVXYZ.LT.100 ) GOTO 40
        INDX = 2
        IVXYZ = JVXYZ - 100
40        IF( JVXYZ.LT.10 ) GOTO 50
          INDY = 2
          JVXYZ = JVXYZ - 10
50        IF( JVXYZ.LT.1 ) GOTO 60
          INDZ = 2
60        JVXYZ = IVXYZ
C       Rotation initialization
C       Determine if initialization is required
70        IF( PHI.EQ.SPHI.AND.THETA.EQ.STHETA ) GOTO 80
C       Compute rotation factors
          SPHI = SINF( 0.0174532925*PHI )
          CPHI = COSF( 0.0174532925*PHI )
          STHETA = SINF( 0.0174532925*THETA )
          CTHETA = COSF( 0.0174532925*THETA )
          A11 = CPHI
          A13 = -SPHI
          A21 = STHETA*SPHI
          A22 = CTHETA
          A23 = STHETA*CPHI
          SPHI = PHI
          STHETA = THETA
C       Processing procedures
C       Set flag to move through the data arrays in the opposite
C       direction
80        INCI = -INCI
C       Set indicator to the first point to be processed
          IF( I.NE.0 ) I = NPNTS + 1
C       Loop to process each point in the data arrays
          DO 530 K=1,NPNTS
C       Data calculations
            I = I + INCI
            GOTO ( 90, 100 ), INDX
90          X = XDATA(1) + (I-1)*XSCALE
```

```
        GOTO 110
100     X = XDATA(I)*XSCALE
110     GOTO ( 120, 130 ), INDY
120     Y = YDATA(I) + (I-1)*YSCALE
        GOTO 140
130     Y = YDATA(I)*YSCALE
140     GOTO ( 150, 160 ), INDZ
150     Z = ZDATA(I) + (NLINE-1)*ZSCALE
        GOTO 170
160     Z = ZDATA(I)*ZSCALE
C     data rotation
170     XXX = A11*X + A13*Z + XREF
        XX = XXX
        IX = IFIX( XX*PIPI + 0.5 )
        YYY = A21*X + A23*Z + YREF
        YY = YYY + A22*Y
        IY = IFIX( YY*PIPI + 0.5 )
C     Restrict figure to plot page
        IF( IX.LE.0 ) IX = 1
        IF( IX.GT.LIMITX ) IX = LIMITX
        IF( IY.LT.10 ) IY = 10
        IF( IY.GT.NYPI ) IY = NYPI
        IF( K.NE.1 ) GOTO 250
C     ( LOC ) is the position  of the previous point with respect to
C     the mask
C             +1     above the mask
C              0     within the limits of the mask
C             -1     below the mask
C     Procedure for initial point of each line
C     Locate initial point with respect to the mask then
C     update the mask
        LOW = IX + IX
        HIGH = LOW + 1
        MLOW = MASK( LOW )
        MHIGH = MASK( HIGH )
        IF( MHIGH-IY ) 200, 210, 180
180     IF( MLOW-IY ) 190, 230, 220
190     LOCOLD = 0
        GOTO 240
200     MASK( HIGH ) = IY
        IF( MLOW.EQ.-1 ) MASK( LOW ) = IY
210     LOCOLD = 1
        GOTO 240
220     MASK( LOW ) = IY
230     LOCOLD = -1
C     Move the raised pen to the initial point
240     CALL IPLOT( IX, IY, 3 )
        JX = IX; JY = IY; IYREF = IY
C     Store verticies if requested
        IF( INDV.EQ.1 ) GOTO 530
        INDEX = INCI + 6
        VERTEX( INDEX ) = XX
        VERTEX(INDEX+1) = YY
        VERTEX(INDEX+8) = XXX
        VERTEX(INDEX+9) = YYY
        IF( NLINE.NE.1 ) GOTO 530
        VERTEX( 1 ) = XX
        VERTEX( 2 ) = YY
        VERTEX( 9 ) = XXX
        VERTEX( 10 ) = YYY
        GOTO 530
C     Special case where change in X co-ordinate is zero
C     A special provision is made at this point so that a line
C     will not mask itself as long as the X coordinate remains
C     constant
250     IF( IX.NE.JX ) GOTO 260
        JY = IY
        GOTO 280
C     compute constants for linear interpolation
260     YINC = FLOAT(IY-JY)/ABS( FLOAT(IX-JX) )
        INCX = (IX-JX)/IABS(IX-JX)
        YJ = JY
C     Perform linear interpolation at each incremental step on
C     the X axis
270     JX = JX + INCX
        YJ = YJ + YINC
        JY = IFIX( YJ + 0.5 )
C     Locate the current point with respect to the mask at that
C     point then plot the increment as a function of the
C     location of the previous point with respect to its mask
        LOW = JX + JX
        HIGH = LOW + 1
        MLOW = MASK( LOW )
        MHIGH = MASK( HIGH )
280     IF( MHIGH-JY ) 300, 300, 290
290     IF( MLOW-JY ) 310, 320, 320
C     The current point is above the mask
300     LOC = +1
        IF( LOCOLD ) 360, 370, 430
```

```
C       The current point is within the mask
310       LOC = 0
          IF( LOCOLD ) 340, 350, 330
C       The current point is below the mask
320       LOC = -1
          IF( LOCOLD ) 510, 450, 440
C       Plotfrom above the mask to within the mask
330       IF( MHIGH.GE.IYREF ) CALL IPLOT( JX, MHIGH, 2 )
          GOTO 350
C       Plot from below the mask to within the mask.
340       IF( MLOW.GE.IYREF ) CALL IPLOT( JX, MLOW, 2)
C       Plot from within the mask to within the mask
350       CALL IPLOT( JX, JY, 3 )
          GOTO 520
C       Plot from below the mask to above the mask
360       IF( MLOW-IYREF ) 370, 380, 380
C       Plot from within the mask to above the mask
370       IF( MHIGH-IYREF ) 400, 390, 390
380       CALL IPLOT( JX, MLOW, 2 )
390       CALL IPLOT( JX, MHIGH, 3 )
          GOTO 430
400       IF( MHIGH.EQ.-1 ) GOTO 430
          OLDHI = HIGH - 2*INCX
          IF( MASK(OLDHI)-JY ) 420, 420, 410
410       CALL IPLOT( JX, JY, 3 )
          GOTO 430
420       CALL IPLOT( JX-INCX, MASK(OLDHI), 3 )
C       Plot from above the mask to above the mask
430       MASK( HIGH ) = JY
          IF( MLOW.EQ.-1 ) MASK( LOW ) = JY
          CALL IPLOT (JX, JY, 2 )
          GOTO520
C       PLOT FROM ABOVE THE MASK TO BELOW THE MASK.
440       IF( MHIGH-IYREF ) 460, 460, 450
C       Plot from within the mask to below the mask
450       IF( MLOW-IYREF ) 470, 470, 480
460       CALL IPLOT( JX, MHIGH, 2 )
470       CALL IPLOT( JX, MLOW, 3 )
          GOTO 510
480       OLDLOW = LOW - 2*INCX
          IF( MASK(OLDLOW)-JY ) 490, 500, 500
490       CALL IPLOT( JX, JY, 3 )
          GOTO 510
500       CALL IPLOT( JX-INCX, MASK(OLDLOW), 3 )
C       Plot from below the mask to below the mask
510       MASK(LOW) = JY
          CALL IPLOT( JX, JY, 2 )
520       IYREF = JY
          LOCOLD = LOC
          IF( JX.NE.IX ) GOTO 270
530       CONTINUE
C       Raise pen
          CALL IPLOT( JX, JY, 3 )
C       Store verticies if requested
          IF( INDV.EQ.1 ) GOTO 540
          INDEX = -INCI + 6
          VERTEX( INDEX ) = XX
          VERTEX(INDEX+1) = YY
          VERTEX(INDEX+8) = XXX
          VERTEX(INDEX+9) = YYY
          IF( NLINE.NE.1 ) GOTO 540
          VERTEX(3) = XX
          VERTEX(4) = YY
          VERTEX(11) = XXX
          VERTEX(12) = YYY
540       I = I - 1
C       Return to calling program
          RETURN
C       Option to modify the masking technique to be used on the
C       following figure so as to plot only above all previous
C       lines.
550       INIT = 0
          RETURN
          END
C
C
          SUBROUTINE FRAMER( IHCOR, VERTEX, MASK )
C       Routine to plot a frame on the projection of a
C       3-dimensional figure as drawn by PLOT3D.
C       Input parameters -
C                IHCOR   -  Number of the vertex of the figure which
C                           appears to be furthest in the background
C                           ( minus Z direction ).
C               VERTEX   -  array containing the coordinates of the
C                           vertices of this figure as returned from
C                           PLOT3D on the last call.
C                 MASK   -  array containing the mask for this figure
C                           as returned by PLOT3D on the last call.
```

```
C       order as their coordinates appear in VERTEX.
C       The MASK array is altered by this routine,
C       but the plotter origin is not moved.
           DIMENSION VERTEX(1), MASK(1), ARRAY(14)
           I = 2*IHDR
           IF( I.LT.2 ) I = 2
           IF( I.GT.8 ) I = 8
C       The vertices which may be hidden
C       are drawn by a call to PLOT3D.
           ARRAY(1) = VERTEX( I-1 )
           ARRAY(8) = VERTEX(I)
           ARRAY(2) = VERTEX(I+7)
           ARRAY(9) = VERTEX(I+8)
           ARRAY(4) = ARRAY( 2 )
           ARRAY(11) = ARRAY(9)
           ARRAY(6) = ARRAY(2)
           ARRAY(13) = ARRAY(9)
           ARRAY(7) = ARRAY(1)
           ARRAY(14) = ARRAY(8)
           I = I - 2
           IF( I.EQ.0 ) I = 8
           ARRAY(3) = VERTEX(I+7)
           ARRAY(10) = VERTEX(I+8)
           I = I + 4
           IF( I.GT.8 ) I = I - 8
           ARRAY(5) = VERTEX(I+7)
           ARRAY(12) = VERTEX(I+8)
           CALL PLOT3D( 110, ARRAY, ARRAY(8), 0.0, 1.0, 1.0, 0.0,
        1 2, 7, 0.0, 0.0, 0.0, 0.0, 0.0, MASK, 0 )
C       The remaining vertices are drawn by calls to PLOT.
           CALL PLOT( VERTEX(I-1), VERTEX(I), 3 )
           I = I - 2
           DO 10 J=1,3
              I = I+2
              IF( I.EQ.10 ) I = 2
              CALL PLOT( VERTEX(I+7), VERTEX(I+8), 2 )
  10       CONTINUE
           CALL PLOT( VERTEX(I-1), VERTEX(I), 2 )
           I = I - 2
           IF( I.EQ.0 ) I = 8
           CALL PLOT( VERTEX(I-1), VERTEX(I), 3 )
           CALL PLOT( VERTEX(I+7), VERTEX(I+8), 2 )
           RETURN
           END
```

```
      SUBROUTINE IPLOT( IX, IY, J )
      IF (J .EQ. 2 ) IVIS = 1
      IF (J .EQ. 3 ) IVIS = 0
      X=IX;Y=IY;
      CALL LINE (X,Y,IVIS)
      RETURN
      END
C
C
      SUBROUTINE PLOT(X ,Y, J)
      DIMENSION W(4)
      DATA W/0.0,10.0,0.0,10.0/
      CALL WINDOW(W)
      IF (J .EQ. 2 ) IVIS = 1
      IF (J .EQ. 3 ) IVIS = 0
      CALL LINER (X,Y,IVIS)
      RETURN
      END
      FUNCTION SINC( X )
      SINC = SIN( X )/X
      RETURN; END
      FUNCTION SINF( X )
      SINF = SIN( X )
      RETURN; END
      FUNCTION COSF( X )
      COSF = COS( X )
      RETURN; END
```

```
C         PROGRAM ORTHO.FOR
C         -----------------
C         THIS PROGRAM GENERATES A BILEVEL
C         PICTURE WITH THE HELP OF ORTHO-
C         GRAPHIC TECHNIQUE.THE PROGRAM
C         PRINTS A 4*4 BINARY PATTERN
C         FOR EACH OF THE 32 GREY LEVELS OF THE
C         INPUT PICTURE WHICH HAS BEEN SAMPLED
C         AND STORED AS A 64*64 MATRIX WHICH IS
C         CALLED IX(I,J).
C         IA(I,J) IS A 32*16 MATRIX WHICH CONTAINS
C         THE 16 BINARY PATTERNS EACH REPEATED TWICE
C         TO CATER FOR 32 GRAY LEVELS.
          DIMENSION IA(32,16),IX(64,64)
          READ (22,*),((IX(I,J),J=1,64),I=1,64)
          READ (23,*),((IA(I,J),J=1,16),I=1,32)
          DO 10I=1,64
          DO 10J=1,64
C         THE SAMPLED PICTURE HAS VALUES FROM 0 TO
C         31 AND THE STORED IA(I,J) MATRIX HAS 1 TO
C         32 .THE SUBSCRIPT L IS USED TO MAKE THEM
C         COMPATIBLE.
          L=IX(I,J)+1
C         THE FILE FOR21.DAT WILL HAVE A 16 BIT VECTOR
C         FOR EACH OF THE 64*64 PIXELS.THE FILE THUS
C         WILL HAVE A ((64*64)*16) MATRIX.
          WRITE (21,*),(IA(L,K),K=1,16)
   10     CONTINUE
          STOP
          END
C         PROGRAM ORTHO1.FOR
C         ------------------
C         THIS PROGRAM READS FROM FOR21.DAT 16 BITS
C         FOR EACH OF THE 64*64 PIXELS AND ARRANGES
C         THEM SEQUENTIALLY AS 4*4 PATTERNS.
C         FOR EACH RUN OF THE OUTER LOOP,THERE ARE
C         4 SMALLER LOOPS WHICH PRINT 256 BITS PER
C         LOOP RESULTING IN 4 ROWS OF 256 COLUMNS
C         EACH REPRESENTING ONE ROW OF THE INPUT
C         PICTURE.THE RESULTING IS STORED AS 0'S
C         AND 1'S IN THE FILE FOR24.DAT.
          DIMENSION A(64,16),B(256)
          INTEGER A,B
          OPEN(UNIT=24,DEVICE='DSKD',FILE='FOR24.DAT')
          DO 5L=1,64
          DO 10I=1,64
          READ(21,*),(A(I,J),J=1,16)
          DO 11J=1,4
          B(4*(I-1)+J)=A(I,J)
   11     CONTINUE
   10     CONTINUE
          WRITE(24,*),(B(I),I=1,256)
          DO 20I=1,64
          DO 21J=5,8
          B(4*(I-1)+J-4)=A(I,J)
   21     CONTINUE
   20     CONTINUE
          WRITE(24,*),(B(I),I=1,256)
          DO 30I=1,64
          DO 31J=9,12
          B(4*(I-1)+J-8)=A(I,J)
   31     CONTINUE
   30     CONTINUE
          WRITE(24,*),(B(I),I=1,256)
          DO 40I=1,64
          DO 41J=13,16
          B(4*(I-1)+J-12)=A(I,J)
   41     CONTINUE
   40     CONTINUE
          WRITE(24,*),(B(I),I=1,256)
    5     CONTINUE
          STOP
          END
C         PROGRAM ORTHO2.FOR
C         ------------------
C         THIS PROGRAM READS THE BINARY DATA FROM
C         FOR24.DAT AND USING THE GPGS SUBROUTINES
C         CONVERTS IT INTO DOTS AND BLANKS ON THE
C         GRAPHICS TERMINAL TEKTRONIX 4006-1.
```

```
      DIMENSION V(4),W1(4)
      DIMENSION V(4),W1(4)
      INTEGER TTY
      INTEGER X(256),BLANK,DOT
      DATA W1/0.,256.,0.,256./,V/0.10,0.55,0.10,0.55/,TTY/1/
      DATA BLANK,DOT/1H ,1H./
      CALL VITDEV(TTY)
      CALL IDEN
      CALL WINDOW(W1(1))
      CALL VPORT(V(1))
      DO 100 I=1,256
      READ(24,*),(X(J),J=1,256)
      DO 100 J=1,256
      IF(X(J).EQ.1)GO TO 100
      X1=I
      Y1=J
      CALL LINE(X1,Y1,0)
      CALL LINE(X1,Y1,1)
100   CONTINUE
      ACCEPT*,OK
      STOP
      END
```

```
      THIS PROGRAM IS FOR FIXED THRESHOLD
      TECHNIQUE.IT READS THE DATA OF THE
      GIVEN PICTURE AND COMPARES IT WITH A
      FIXED THRESHOLD OF 16.
      DIMENSION V(4),W1(4)
      INTEGER TTY
      INTEGER X(128,128),Y(128,128)
      DATA W1/0.,128.,0.,128./,V/0.50,0.80,0.50,0.80/,TTY/1/
      CALL WIPDEV(TTY)
      CALL IDEN
      CALL WINDOW(W1(1))
      CALL VPORT(V(1))
      READ*,((X(I,J),J=1,128),I=1,128)
      DO 10I=1,128
      DO 10J=1,128
      IF(X(I,J).GT.16)GO TO 10
      X1=I
      Y1=J
      CALL LINE(X1,Y1,0)
      CALL LINE(X1,Y1,1)
10    CONTINUE
      ACCEPT*,OK
      STOP
      END
```

```
C       PROGRAM FOR THE GENERATION OF
C       256*256 PICTURE OF
C       LINCOLN BY ORDERED DITHER
        DIMENSION V(4),W1(4)
        INTEGER TTY
        INTEGER X(256),Y(256),BLANK,DOT
C       OPEN(UNIT=22,DEVICE=DSKD,FILE='FOR22.DAT')
C       OPEN(UNIT=23,DEVICE=DSKD,FILE='FOR23.DAT')
        DATA W1/0.,256.,0.,256./,V/0.,0.25,0.,0.25/,TTY/1/
        DATA BLANK,DOT/1H ,1H./
        CALL NITDEV(TTY)
        CALL IDEN
        CALL WINDW(W1(1))
        CALL VPORT(V(1))
        DO 100 I=1,256
        READ(24,*),(X(J),J=1,256)
        READ(23,*),(Y(J),J=1,256)
        DO 100 J=1,256
        IF(X(J).GT.Y(J))GO TO 100
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
  100   CONTINUE
C       PRINT 110
C  110  FORMAT(//20X,'THE BILEVEL PICTURE IS'/30X,'-----------')
C       PRINT120,((Z(I,J),J=1,64),I=1,64)
C  120  FORMAT(5X,64A1)
        ACCEPT*,OK
        STOP
        END
```

```
C           PROGRAM CONAVE.FOR
C           ------------------
C           THIS PROGRAM GENERATES A BILEVEL
C           PICTURE BY THE CONSTRAINED AVERAGE
C           METHOD.THE OUTPUT PICURE IS A
C   64*64   BILEVEL PICTURE
C           PRINTED ON THE LINE PRINTER.
            INTEGER I,J,W(64,64),X(64,64),Y(64,64),Z(64,64),BLANK,DOT
            DATA BLANK,DOT/1H ,1H./
            READ*,((X(I,J),J=1,64),I=1,64)
C           PRINTING OF THE MULTILEVEL PICTURE DATA.
            PRINT 10
   10       FORMAT(//80X,'THE PICTURE MATRIX IS',/80X,'---------------
           1------')
            DO 20I=1,64
   20       PRINT 30,(X(I,J),J=1,64)
   30       FORMAT(5X,64I2)
C           CALCULATING THE AVERAGE INTENSITY BY TAKING EXPLICIT SUM
C           OVER THE IMMEDIATE NEIGHBORHOOD.
            DO 40J=1,64
            W(1,J)=X(1,J)
   40       CONTINUE
            DO 50I=2,63
            W(I,1)=X(I,1)
            DO 60J=2,63
            W(I,J)=FLOAT(X(I-1,J-1)+X(I-1,J)+X(I-1,J+1)
           1+X(I,J-1)+X(I,J)+X(I,J+1)+X(I+1,J-1)+X(I+1,J)
           1+X(I+1,J+1))/9+0.5
   60       CONTINUE
            W(I,64)=X(I,64)
   50       CONTINUE
            DO 70J=1,64
            W(64,J)=X(64,J)
   70       CONTINUE
C           FORMING OF THE  THRESHOLD MATRIX
            DO 300I=1,64
            DO 290J=1,64
            Y(I,J)=(2.+((1.-4./31.0*W(I,J)))
  290       CONTINUE
  300       CONTINUE
C           PRINTING OF THRESHOLD MATRIX
            PRINT 310
  310       FORMAT(//80X,'THE THRESHOLD MATRIX IS',/80X,'---------------
           1---------')
            DO 320I=1,64
  320       PRINT 330,(W(I,J),J=1,64)
  330       FORMAT(5X,64I2)
C           COMPARISION OF INTENSITIES WITH MODIFIED THRESHOLDS AND
C           FORMING OF THE BILEVEL PICTURE
            DO 400I=1,64
            DO 390J=1,64
            IF(X(I,J).LE.W(I,J))GO TO 380
            CONTINUE
            Z(I,J)=BLANK
            GO TO 390
  380       Z(I,J)=DOT
  390       CONTINUE
  400       CONTINUE
C           PRINTING OF THE BILEVEL PICTURE
            PRINT 410
  410       FORMAT(//80X,'THE BILEVEL PICTURE IS',/80X,'---------------
           1-------')
            PRINT 420,((Z(I,J),J=1,64),I=1,64)
  420       FORMAT(5X,64A1)
            STOP
            END
```

```
      DIMENSION V(4),W1(4)
      INTEGER TTY
      INTEGER I,J,X(128,128),Y(128,128),W
      DATA W1/0.,128.,0.,128./,V/0.20,0.50,0.20,0.50/,TTY/1/
      CALL NITDEV(TTY)
      CALL IDEN
      CALL WINDOW(W1(1))
      CALL VPORT(V(1))
      READ*,((X(I,J),J=1,128),I=1,128)
C     CALCULATING THE AVERAGE INTENSITY BY TAKING EXPLICIT SUM
C     OVER NEIGHBORHOOD
      DO 40J=1,128
      Y(1,J)=X(1,J)
   40 CONTINUE
      DO 50I=2,127
      Y(I,1)=X(I,1)
      DO 60J=2,127
      W=FLOAT(X(I-1,J-1)+X(I-1,J)+X(I-1,J+1)
     1+X(I,J-1)+X(I,J)+X(I,J+1)+X(I+1,J-1)+X(I+1,J)
     1+X(I+1,J+1))/9+0.5
      Y(I,J)=(2.+(1.-4./31.)*W)
   60 CONTINUE
      Y(I,128)=X(I,128)
   50 CONTINUE
      DO 70J=1,128
      Y(128,J)=X(128,J)
   70 CONTINUE
C     FORMING OF THE BILEVEL PICTURE
      DO 400I=1,128
      DO 390J=1,128
      IF(X(I,J).LT.X(I,J))GO TO 380
      X1=I
      Y1=J
      CALL LINE (X1,Y1,0)
      CALL LINE (X1,Y1,1)
  380 CONTINUE
  390 CONTINUE
  400 CONTINUE
      ACCEPT*,OK
      STOP
      END
```

```
C       PICTURE BY ERROR DIFFUSION TECHNIQUE
        DIMENSION V(4),W1(4)
        INTEGER TTY
        INTEGER X(128,128),Y(128,128),E(128,128),Z(128,128),BLANK,DOT
        DATA W1/0.,128.,0.,128./,V/0.25,0.55,0.25,0.55/,TTY/1/
        DATA BLANK,DOT/1H ,1H./
        CALL NITDEV(TTY)
        CALL IDEN
        CALL WINDOW(W1(1))
        CALL VPORT(V(1))
        READ*, ((X(I,J),J=1,128),I=1,128)
        DO 30J=1,128
        E(1,J)=0
        Y(1,J)=X(1,J)
        IF(Y(1,J).GT.15)GO TO 30
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
30      CONTINUE
        DO 40J=1,128
        E(2,J)=0
        Y(2,J)=X(2,J)
        IF(Y(2,J).GT.15)GO TO 40
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
40      CONTINUE
              DO 50I=3,128
        Y(I,1)=X(I,1)
        E(I,1)=0
        IF(Y(I,1).GT.15)GO TO 41
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
41      CONTINUE
        Y(I,2)=X(I,2)
        E(I,2)=0
        IF(Y(I,2).GT.15)GO TO 43
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
43      CONTINUE
        DO 60J=3,126
        Y(I,J)=X(I,J)+(0.15*E(I,J-1)+0.1*E(I,J-2)+0.06*E(I-1,J+2)
       1+0.1*E(I-1,J+1)+0.15*E(I-1,J)+0.1*E(I-1,J-1)+0.06*E(I-1,J-2)
       1+0.03*E(I-2,J+2)+0.06*E(I-2,J+1)+0.1*E(I-2,J)+0.06*E(I-2,J-1)
       1+0.03*E(I-2,J-2))+0.5
        IF(Y(I,J).GT.15)GO TO 62
        E(I,J)=Y(I,J)
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
        GO TO 60
62      CONTINUE
        E(I,J)=Y(I,J)-31
60      CONTINUE
        Y(I,127)=X(I,127)
        E(I,127)=0
        IF(Y(I,127).GT.15)GO TO 63
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
63      CONTINUE
        Y(I,128)=X(I,128)
        E(I,128)=0
        IF(Y(I,128).GT.15)GO TO 65
        X1=I
        Y1=J
        CALL LINE(X1,Y1,0)
        CALL LINE(X1,Y1,1)
65      CONTINUE
50      CONTINUE
        ACCEPT*,OK
        STOP
        END
```

THE INPUT PATTERN IS

```
2.397895    ,   2.397895    ,   2.397895    ,   2.397895    ,   2.397895    ,   2.397895    ,   2.397895    ,   2.3
2.397895
       THE SYSTEM RESPONSE IS
       2.13  4.29  6.48  6.58  6.68  6.78  6.88  6.98  7.08  7.19  7.29  7.33  7.29  7.19  7.08  6.98  6.88  6.78  6.6
 4.29  2.13
       4.29  8.64 13.05 13.25 13.45 13.65 13.85 14.06 14.27 14.48 14.70 14.77 14.70 14.48 14.27 14.06 13.85 13.65 13.4
 8.64  4.29
       6.48 13.05 19.73 20.03 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.32 22.21 21.89 21.57 21.25 20.94 20.63 20.3
13.05  6.48
       6.58 13.25 20.03 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.65 22.54 22.21 21.89 21.57 21.25 20.94 20.6
13.25  6.58
       6.68 13.45 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.87 22.98 22.87 22.54 22.21 21.89 21.57 21.25 20.9
13.45  6.68
       6.78 13.65 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.33 23.21 22.87 22.54 22.21 21.89 21.57 21.2
13.65  6.78
       6.88 13.85 20.94 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.67 23.56 23.21 22.87 22.54 22.21 21.89 21.5
13.85  6.88
       6.98 14.06 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.02 23.91 23.56 23.21 22.87 22.54 22.21 21.8
14.06  6.98
       7.08 14.27 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.25 24.38 24.25 23.91 23.56 23.21 22.87 22.54 22.2
14.27  7.08
       7.19 14.48 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.25 24.62 24.74 24.62 24.25 23.91 23.56 23.21 22.87 22.5
14.48  7.19
       7.29 14.70 22.21 22.54 22.87 23.21 23.56 23.91 24.26 24.62 24.98 25.10 24.98 24.62 24.26 23.91 23.56 23.21 22.8
14.70  7.29
       7.33 14.77 22.32 22.65 22.99 23.33 23.67 24.02 24.38 24.74 25.10 25.23 25.10 24.74 24.38 24.02 23.67 23.33 22.9
14.77  7.33
       7.29 14.70 22.21 22.54 22.87 23.21 23.56 23.91 24.26 24.62 24.98 25.10 24.98 24.62 24.26 23.91 23.56 23.21 22.8
14.70  7.29
       7.19 14.48 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.26 24.62 24.74 24.62 24.26 23.91 23.56 23.21 22.87 22.5
14.48  7.19
       7.08 14.27 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.25 24.38 24.25 23.91 23.56 23.21 22.87 22.54 22.2
14.27  7.08
       6.98 14.06 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.91 24.02 23.91 23.56 23.21 22.87 22.54 22.21 21.8
14.06  6.98
       6.88 13.85 20.94 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.56 23.67 23.56 23.21 22.87 22.54 22.21 21.89 21.5
13.85  6.88
       6.78 13.65 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.87 23.21 23.33 23.21 22.87 22.54 22.21 21.89 21.57 21.2
13.65  6.78
       6.68 13.45 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.87 22.98 22.87 22.54 22.21 21.89 21.57 21.25 20.9
13.45  6.68
       6.58 13.25 20.03 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.54 22.65 22.54 22.21 21.89 21.57 21.25 20.94 20.6
13.25  6.58
       6.48 13.05 19.73 20.03 20.33 20.63 20.94 21.25 21.57 21.89 22.21 22.32 22.21 21.89 21.57 21.25 20.94 20.63 20.3
13.05  6.48
       4.29  8.64 13.05 13.25 13.45 13.65 13.85 14.06 14.27 14.48 14.70 14.77 14.70 14.48 14.27 14.06 13.85 13.65 13.4
 8.64  4.29
       2.13  4.29  6.48  6.58  6.68  6.78  6.88  6.98  7.08  7.19  7.29  7.33  7.29  7.19  7.08  6.98  6.88  6.78  6.6
 4.29  2.13
```

THE INPUT PATTERN IS

2.397895 , 2.397895 , 2.397895 , 2.397895 , 0.6931472 , 2.397895 , 2.397895 ,
2.397895 ,

THE SYSTEM RESPONSE IS

	2.13	4.29	6.48	6.58	6.68	6.78	6.88	6.98	7.08	7.19	7.29	7.33	7.29	7.19	7.08	6.98	6.88	6.78
4.29	2.13																	
	4.29	7.13	11.52	11.69	11.87	12.05	12.23	12.41	12.59	12.78	12.97	13.02	12.97	12.78	12.59	12.41	12.23	12.05
7.13	4.29																	
	6.48	11.52	18.18	18.45	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.54	20.46	20.16	19.86	19.57	19.28	19.00
11.52	6.48																	
	6.58	11.69	18.45	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.76	20.84	20.76	20.46	20.16	19.86	19.57	19.28
11.69	6.58																	
	6.68	11.87	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.15	21.07	20.76	20.46	20.16	19.86	19.57
11.87	6.68																	
	6.78	12.05	19.00	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.47	21.38	21.07	20.76	20.46	20.16	19.86
12.05	6.78																	
	6.88	12.23	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.70	21.78	21.70	21.38	21.07	20.76	20.46	20.16
12.23	6.88																	
	6.98	12.41	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.11	22.02	21.70	21.38	21.07	20.76	20.46
12.41	6.98																	
	7.08	12.59	19.86	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.43	22.34	22.02	21.70	21.38	21.07	20.76
12.59	7.08																	
	7.19	12.78	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.67	22.76	22.67	22.34	22.02	21.70	21.38	21.07
12.78	7.19																	
	7.29	12.97	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.67	23.01	23.10	23.01	22.67	22.34	22.02	21.70	21.38
12.97	7.29																	
	7.33	13.02	20.54	20.84	21.15	21.47	21.78	22.11	22.43	22.76	23.10	23.19	23.10	22.76	22.43	22.11	21.78	21.47
13.02	7.33																	
	7.29	12.97	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.67	23.01	23.10	23.01	22.67	22.34	22.02	21.70	21.38
12.97	7.29																	
	7.19	12.78	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.67	22.76	22.67	22.34	22.02	21.70	21.38	21.07
12.78	7.19																	
	7.08	12.59	19.86	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.34	22.43	22.34	22.02	21.70	21.38	21.07	20.76
12.59	7.08																	
	6.98	12.41	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.70	22.02	22.11	22.02	21.70	21.38	21.07	20.76	20.46
12.41	6.98																	
	6.88	12.23	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.70	21.78	21.70	21.38	21.07	20.76	20.46	20.16
12.23	6.88																	
	6.78	12.05	19.00	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.38	21.47	21.38	21.07	20.76	20.46	20.16	19.86
12.05	6.78																	
	6.68	11.87	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.76	21.07	21.15	21.07	20.76	20.46	20.16	19.86	19.57
11.87	6.68																	
	6.58	11.69	18.45	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.76	20.84	20.76	20.46	20.16	19.86	19.57	19.28
11.69	6.58																	
	6.48	11.52	18.18	18.45	18.72	19.00	19.28	19.57	19.86	20.16	20.46	20.54	20.46	20.16	19.86	19.57	19.28	19.00
11.52	6.48																	
	4.29	7.13	11.52	11.69	11.87	12.05	12.23	12.41	12.59	12.78	12.97	13.02	12.97	12.78	12.59	12.41	12.23	12.05
7.13	4.29																	
	2.13	4.29	6.48	6.58	6.68	6.78	6.88	6.98	7.08	7.19	7.29	7.33	7.29	7.19	7.08	6.98	6.88	6.78
4.29	2.13																	

THE SYSTEM RESPONSE IS

	2.13	4.29	5.48	5.58	6.58	6.78	6.88	6.98	7.08	7.19	7.29	7.33	7.29	7.19	7.08	6.98	6.88	6.78	6.
4.29	2.13																		
	4.29	7.13	10.01	10.15	10.31	10.46	10.62	10.78	10.94	11.10	11.27	11.29	11.21	11.05	10.89	10.73	10.57	10.42	10.
5.59	2.78																		
	6.48	11.62	16.54	16.89	17.14	17.40	17.65	17.92	18.18	18.45	18.73	18.78	18.58	18.40	18.13	17.87	17.61	17.35	17.
9.95	4.95																		
	6.58	11.69	16.89	17.14	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.06	18.95	18.68	18.40	18.13	17.87	17.61	17.
10.11	5.02																		
	6.68	11.87	17.14	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.35	19.23	18.95	18.68	18.40	18.13	17.87	17.
10.25	5.09																		
	6.78	12.05	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.63	19.52	19.23	18.95	18.68	18.40	18.13	17.
10.42	5.17																		
	6.88	12.23	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.86	19.92	19.81	19.52	19.23	18.95	18.68	18.40	18.
10.57	5.25																		
	6.98	12.41	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.86	20.15	20.22	20.10	19.81	19.52	19.23	18.95	18.68	18.
10.73	5.32																		
	7.08	12.59	18.18	18.45	18.73	19.01	19.29	19.57	19.86	20.15	20.45	20.52	20.40	20.10	19.81	19.52	19.23	18.95	18.
10.89	5.40																		
	7.19	12.78	18.45	18.73	19.01	19.29	19.57	19.86	20.16	20.46	20.75	20.82	20.71	20.40	20.10	19.81	19.52	19.23	18.
11.05	5.48																		
	7.29	12.97	18.73	19.01	19.29	19.57	19.86	20.16	20.46	20.75	21.06	21.13	21.01	20.70	20.40	20.10	19.81	19.52	19.
11.21	5.57																		
	7.33	13.02	18.78	19.05	19.35	19.63	19.92	20.22	20.52	20.82	21.13	21.19	21.07	20.75	20.45	20.16	19.87	19.58	19.
11.23	5.58																		
	7.29	12.97	18.73	19.01	19.29	19.57	19.86	20.16	20.46	20.75	21.06	21.13	21.01	20.70	20.40	20.10	19.81	19.52	19.
11.21	5.57																		
	7.19	12.78	18.45	18.73	19.01	19.29	19.57	19.86	20.16	20.46	20.75	20.82	20.71	20.40	20.10	19.81	19.52	19.23	18.
11.05	5.48																		
	7.08	12.59	18.18	18.45	18.73	19.01	19.29	19.57	19.86	20.15	20.45	20.52	20.40	20.10	19.81	19.52	19.23	18.95	18.
10.89	5.40																		
	6.98	12.41	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.86	20.15	20.22	20.10	19.81	19.52	19.23	18.95	18.68	18.
10.73	5.32																		
	6.88	12.23	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.86	19.92	19.81	19.52	19.23	18.95	18.68	18.40	18.
10.57	5.25																		
	6.78	12.05	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.57	19.63	19.52	19.23	18.95	18.68	18.40	18.13	17.
10.42	5.17																		
	6.68	11.87	17.14	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.29	19.35	19.23	18.95	18.68	18.40	18.13	17.87	17.
10.25	5.09																		
	6.58	11.69	16.89	17.14	17.40	17.65	17.92	18.18	18.45	18.73	19.01	19.06	18.95	18.68	18.40	18.13	17.87	17.61	17.
10.11	5.02																		
	6.48	11.62	16.54	16.89	17.14	17.40	17.65	17.92	18.18	18.45	18.73	18.78	18.58	18.40	18.13	17.87	17.61	17.35	17.
9.95	4.95																		
	4.29	7.13	10.01	10.15	10.31	10.46	10.62	10.78	10.94	11.10	11.27	11.29	11.21	11.05	10.89	10.73	10.57	10.42	10.
5.59	2.78																		
	2.13	4.29	5.48	5.58	6.58	6.78	6.88	6.98	7.08	7.19	7.29	7.33	7.29	7.19	7.08	6.98	6.88	6.78	6.
4.29	2.13																		

THE INPUT PATTERN IS

2.397895 , 2.397895 , 2.397895 , 2.397895 , 0.6931472 , 0.6931472 , 2.397895 ,
2.397895

THE SISTEM RESPONSE IS

	2.13	4.29	6.48	6.58	6.68	6.78	6.88	6.98	7.08	7.19	7.29	7.33	7.29	7.19	7.08	6.98	6.88	6.78
4.29	2.13																	
	4.29	7.13	10.01	10.16	10.31	10.46	10.62	10.78	10.94	11.10	11.27	11.29	11.21	11.05	10.89	10.73	10.57	10.42
5.59	2.78																	
	6.48	10.01	15.10	15.33	15.56	15.79	16.02	16.26	16.51	16.75	17.00	17.03	16.95	16.70	16.45	16.21	15.98	15.74
8.45	4.95																	
	6.58	10.16	15.33	15.56	15.79	16.02	16.26	16.51	16.75	17.00	17.25	17.28	17.20	16.95	16.70	16.45	16.21	15.98
8.58	5.02																	
	6.68	10.31	15.56	15.79	16.02	16.26	16.51	16.75	17.00	17.25	17.51	17.54	17.45	17.20	16.95	16.70	16.45	16.21
8.70	5.09																	
	6.78	10.46	15.79	16.02	16.26	16.51	16.75	17.00	17.25	17.51	17.77	17.80	17.71	17.45	17.20	16.95	16.70	16.45
8.83	5.17																	
	6.88	10.62	16.02	16.26	16.51	16.75	17.00	17.25	17.51	17.77	18.03	18.06	17.98	17.71	17.45	17.20	16.95	16.70
8.97	5.25																	
	6.98	10.78	16.26	16.51	16.75	17.00	17.25	17.51	17.77	18.03	18.30	18.33	18.24	17.98	17.71	17.45	17.20	16.95
9.10	5.32																	
	7.08	10.94	16.51	16.75	17.00	17.25	17.51	17.77	18.03	18.30	18.57	18.60	18.51	18.24	17.98	17.71	17.45	17.20
9.24	5.40																	
	7.19	11.10	16.75	17.00	17.25	17.51	17.77	18.03	18.30	18.57	18.84	18.88	18.78	18.51	18.24	17.98	17.71	17.45
9.37	5.48																	
	7.29	11.27	17.00	17.25	17.51	17.77	18.03	18.30	18.57	18.84	19.12	19.15	19.06	18.78	18.51	18.24	17.98	17.71
9.51	5.57																	
	7.33	11.29	17.03	17.28	17.54	17.80	18.06	18.33	18.60	18.88	19.15	19.19	19.10	18.82	18.54	18.27	18.01	17.74
9.51	5.58																	
	7.29	11.21	16.95	17.20	17.45	17.71	17.98	18.24	18.51	18.78	19.06	19.10	19.00	18.73	18.45	18.19	17.92	17.66
9.46	5.57																	
	7.19	11.05	16.70	16.95	17.20	17.45	17.71	17.98	18.24	18.51	18.78	18.82	18.73	18.45	18.19	17.92	17.66	17.40
9.32	5.48																	
	7.08	10.89	16.45	16.70	16.95	17.20	17.45	17.71	17.98	18.24	18.51	18.54	18.45	18.19	17.92	17.66	17.40	17.15
9.19	5.40																	
	6.98	10.73	16.21	16.45	16.70	16.95	17.20	17.45	17.71	17.98	18.24	18.27	18.19	17.92	17.66	17.40	17.15	16.90
9.05	5.32																	
	6.88	10.57	15.98	16.21	16.45	16.70	16.95	17.20	17.45	17.71	17.98	18.01	17.92	17.66	17.40	17.15	16.90	16.65
8.92	5.25																	
	6.78	10.42	15.74	15.98	16.21	16.45	16.70	16.95	17.20	17.45	17.71	17.74	17.66	17.40	17.15	16.90	16.65	16.40
8.79	5.17																	
	6.68	10.26	15.51	15.74	15.98	16.21	16.45	16.70	16.95	17.20	17.45	17.49	17.40	17.15	16.90	16.65	16.40	16.16
8.66	5.09																	
	6.58	10.11	15.28	15.51	15.74	15.98	16.21	16.45	16.70	16.95	17.20	17.23	17.15	16.90	16.65	16.40	16.16	15.93
8.53	5.02																	
	6.48	9.96	15.06	15.29	15.51	15.74	15.98	16.21	16.45	16.70	16.95	16.98	16.90	16.65	16.40	16.16	15.93	15.69
8.41	4.95																	
	4.29	5.59	8.45	8.58	8.70	8.83	8.97	9.10	9.24	9.37	9.51	9.51	9.46	9.32	9.19	9.05	8.92	8.79
4.06	2.78																	
	2.13	2.78	4.95	5.02	5.09	5.17	5.25	5.32	5.40	5.48	5.57	5.58	5.57	5.48	5.40	5.32	5.25	5.17
2.78	2.13																	

THE SYSTEM RESPONSE IS

	2.13	4.29	4.97	5.04	5.12	5.19	5.27	5.35	5.43	5.51	5.59	5.60	5.51	5.49	5.38	5.30	5.22	5.15	5.
2.75	0.62																		
	2.78	7.11	8.45	8.58	8.70	8.83	8.97	9.10	9.24	9.37	9.51	9.56	9.45	9.32	9.18	9.05	8.92	8.79	8.
5.55	1.24																		
	4.95	9.94	13.52	13.72	13.93	14.14	14.35	14.55	14.78	15.00	15.22	15.28	15.17	14.94	14.73	14.51	14.30	14.09	13.
8.43	3.39																		
	5.02	10.14	13.72	13.93	14.14	14.35	14.55	14.73	15.00	15.22	15.44	15.50	15.39	15.17	14.94	14.73	14.51	14.30	14.
8.53	3.44																		
	5.09	10.29	13.93	14.14	14.35	14.55	14.73	15.00	15.22	15.44	15.67	15.73	15.52	15.34	15.17	14.94	14.73	14.51	14.
8.65	3.49																		
	5.17	10.44	14.14	14.35	14.56	14.78	15.00	15.22	15.44	15.67	15.91	15.97	15.75	15.52	15.39	15.17	14.94	14.73	14.
8.79	3.54																		
	5.25	10.60	14.35	14.55	14.78	15.00	15.22	15.44	15.67	15.91	15.14	15.20	15.39	15.45	15.62	15.39	15.17	14.94	14.
8.92	3.59																		
	5.32	10.75	14.56	14.78	15.00	15.22	15.11	15.57	15.91	16.13	15.38	16.44	16.37	15.09	15.65	15.62	15.09	15.17	14.
9.05	3.65																		
	5.40	10.91	14.78	15.00	15.22	15.19	15.57	15.91	16.19	15.34	15.62	16.69	15.57	15.82	16.09	15.85	15.62	15.39	15.
9.18	3.70																		
	5.48	11.08	15.00	15.22	15.44	15.57	15.91	16.24	16.38	15.62	15.87	15.93	15.81	15.57	16.32	16.09	15.85	15.62	15.
9.32	3.75																		
	5.57	11.24	15.22	15.44	15.57	15.91	15.14	16.33	16.62	16.87	17.12	17.18	17.35	15.81	16.57	16.32	16.09	15.85	15.
9.45	3.81																		
	5.58	11.25	15.28	15.50	15.73	15.96	15.20	15.44	15.58	16.93	17.18	17.25	17.12	15.87	16.63	16.39	16.15	15.91	15.
9.51	3.85																		
	5.57	11.49	15.22	15.44	15.57	15.91	15.14	16.38	15.62	15.87	17.12	17.18	17.05	15.81	16.57	16.33	16.09	15.85	15.
9.45	3.85																		
	5.48	11.03	15.00	15.22	15.44	15.57	15.91	15.14	15.38	16.62	15.87	15.93	15.81	15.57	16.33	16.09	15.85	15.62	15.
9.32	3.81																		
	5.40	10.85	14.78	15.00	15.22	15.44	15.57	15.91	16.14	15.38	16.62	16.68	15.57	15.33	16.09	15.85	15.62	15.39	15.
9.19	3.75																		
	5.32	10.70	14.56	14.73	15.00	15.22	15.44	15.67	15.91	15.14	15.38	15.44	15.33	15.07	15.85	15.62	15.39	15.17	14.
9.05	3.70																		
	5.25	10.55	14.35	14.55	14.78	15.00	15.22	15.44	15.67	15.91	16.14	16.20	15.09	15.35	15.62	15.39	15.17	14.95	14.
8.92	3.64																		
	5.17	10.39	14.14	14.35	14.56	14.78	15.00	15.22	15.44	15.67	15.91	15.96	15.55	15.52	15.39	15.17	14.95	14.73	14.
8.79	3.59																		
	5.09	10.24	13.93	14.14	14.35	14.56	14.73	15.00	15.22	15.44	15.67	15.73	15.52	15.39	15.17	14.95	14.73	14.51	14.
8.66	3.54																		
	5.02	10.09	13.72	13.93	14.14	14.35	14.55	14.78	15.00	15.22	15.44	15.50	15.39	15.17	14.95	14.73	14.51	14.30	14.
8.53	3.48																		
	4.95	9.94	13.52	13.72	13.93	14.14	14.35	14.55	14.78	15.00	15.22	15.28	15.17	14.95	14.73	14.51	14.30	14.09	13.
8.41	3.43																		
	2.78	5.57	8.33	8.55	8.58	8.81	8.94	9.08	9.21	9.35	9.49	9.53	9.49	9.35	9.21	9.08	8.94	8.81	8.
5.57	2.73																		
	2.13	2.78	4.95	5.02	5.09	5.17	5.25	5.32	5.40	5.48	5.57	5.58	5.57	5.48	5.40	5.32	5.25	5.17	5.
2.73	2.13																		

THE SYSTEM RESPONSE IS

	2.13	2.78	4.95	5.02	5.09	5.17	5.25	5.32	5.40	5.48	5.57	5.58	5.57	5.48	5.40	5.32	5.25	5.17	5.
2.78	2.13																		
	2.78	4.95	5.89	7.00	7.10	7.21	7.31	7.42	7.53	7.64	7.75	7.78	7.76	7.64	7.53	7.42	7.31	7.21	7.
4.95	2.78																		
	4.95	6.89	11.94	12.12	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.47	13.44	13.24	13.05	12.86	12.67	12.48	12.
6.89	4.95																		
	5.02	7.00	12.12	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.67	13.64	13.44	13.24	13.05	12.86	12.67	12.
7.00	5.02																		
	5.09	7.10	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.84	13.87	13.84	13.64	13.44	13.24	13.05	12.86	12.
7.10	5.09																		
	5.17	7.21	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.08	14.05	13.84	13.64	13.44	13.24	13.05	12.
7.21	5.17																		
	5.25	7.31	12.67	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.29	14.25	14.05	13.84	13.64	13.44	13.24	13.
7.31	5.25																		
	5.32	7.42	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.50	14.46	14.25	14.05	13.84	13.64	13.44	13.
7.42	5.32																		
	5.40	7.53	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.71	14.68	14.46	14.25	14.05	13.84	13.64	13.
7.53	5.40																		
	5.48	7.64	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.90	14.93	14.90	14.68	14.46	14.25	14.05	13.84	13.
7.64	5.48																		
	5.57	7.75	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.90	15.12	15.15	15.12	14.90	14.68	14.46	14.25	14.05	13.
7.75	5.57																		
	5.58	7.78	13.47	13.67	13.87	14.08	14.29	14.50	14.71	14.93	15.15	15.18	15.15	14.93	14.71	14.50	14.29	14.08	13.
7.78	5.58																		
	5.57	7.76	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.90	15.12	15.15	15.12	14.90	14.68	14.46	14.25	14.05	13.
7.76	5.57																		
	5.48	7.64	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.90	14.93	14.90	14.68	14.46	14.25	14.05	13.84	13.
7.64	5.48																		
	5.40	7.53	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.68	14.71	14.68	14.46	14.25	14.05	13.84	13.64	13.
7.53	5.40																		
	5.32	7.42	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.46	14.50	14.46	14.25	14.05	13.84	13.64	13.44	13.
7.42	5.32																		
	5.25	7.31	12.67	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.25	14.29	14.25	14.05	13.84	13.64	13.44	13.24	13.
7.31	5.25																		
	5.17	7.21	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.84	14.05	14.08	14.05	13.84	13.64	13.44	13.24	13.05	12.
7.21	5.17																		
	5.09	7.10	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.84	13.87	13.84	13.64	13.44	13.24	13.05	12.86	12.
7.10	5.09																		
	5.02	7.00	12.12	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.64	13.67	13.64	13.44	13.24	13.05	12.86	12.67	12.
7.00	5.02																		
	4.95	6.89	11.94	12.12	12.30	12.48	12.67	12.86	13.05	13.24	13.44	13.47	13.44	13.24	13.05	12.86	12.67	12.48	12.
6.89	4.95																		
	2.78	4.95	5.89	7.00	7.10	7.21	7.31	7.42	7.53	7.64	7.75	7.78	7.76	7.64	7.53	7.42	7.31	7.21	7.
4.95	2.78																		
	2.13	2.78	4.95	5.02	5.09	5.17	5.25	5.32	5.40	5.48	5.57	5.58	5.57	5.48	5.40	5.32	5.25	5.17	5.
2.78	2.13																		

THE INPUT PATTERN IS

0.6931472 , 0.6931472 , 2.397895 , 2.397895 , 0.6931472 , 0.6931472 , 2.397895 ,
0.6931472

THE SYSTEM RESPONSE IS

	0.62	1.24	3.39	3.44	3.49	3.54	3.59	3.65	3.70	3.76	3.81	3.85	3.86	3.81	3.75	3.70	3.64	3.59
2.79	2.13																	
	2.75	4.03	6.67	6.97	7.08	7.18	7.29	7.40	7.51	7.62	7.73	7.75	7.73	7.62	7.51	7.40	7.29	7.18
4.06	2.78																	
	4.92	6.87	10.40	10.56	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.71	11.66	11.49	11.32	11.15	10.99	10.83
5.36	3.43																	
	5.00	6.97	10.56	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.88	11.89	11.83	11.66	11.49	11.32	11.15	10.99
5.44	3.49																	
	5.07	7.08	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.06	12.01	11.83	11.66	11.49	11.32	11.15
5.52	3.54																	
	5.15	7.18	10.88	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.24	12.19	12.01	11.83	11.66	11.49	11.32
5.60	3.59																	
	5.22	7.29	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.42	12.37	12.19	12.01	11.83	11.66	11.49
5.68	3.64																	
	5.30	7.40	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.61	12.55	12.37	12.19	12.01	11.83	11.66
5.77	3.70																	
	5.38	7.51	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.80	12.73	12.55	12.37	12.19	12.01	11.83
5.85	3.75																	
	5.46	7.62	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.98	12.98	12.92	12.73	12.55	12.37	12.19	12.01
5.94	3.81																	
	5.54	7.73	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.98	13.17	13.18	13.11	12.92	12.73	12.55	12.37	12.19
6.03	3.86																	
	5.60	7.80	11.77	11.94	12.12	12.30	12.48	12.67	12.85	13.04	13.24	13.24	13.18	12.98	12.80	12.61	12.42	12.24
6.07	3.85																	
	5.59	7.78	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.98	13.17	13.19	13.11	12.92	12.73	12.55	12.36	12.18
5.98	3.81																	
	5.61	7.67	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.98	12.98	12.92	12.73	12.55	12.36	12.18	12.01
5.89	3.76																	
	5.43	7.56	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.79	12.80	12.73	12.55	12.36	12.18	12.01	11.83
5.80	3.70																	
	5.35	7.45	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.60	12.61	12.55	12.36	12.18	12.01	11.83	11.66
5.72	3.65																	
	5.27	7.31	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.42	12.42	12.36	12.18	12.01	11.83	11.66	11.49
5.63	3.59																	
	5.19	7.23	10.88	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.24	12.24	12.18	12.01	11.83	11.66	11.49	11.32
5.55	3.54																	
	5.12	7.12	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.88	12.06	12.06	12.01	11.83	11.66	11.49	11.32	11.15
5.47	3.49																	
	5.04	7.02	10.55	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.88	11.89	11.83	11.66	11.49	11.32	11.15	10.99
5.39	3.44																	
	4.97	6.92	10.40	10.56	10.72	10.88	11.04	11.20	11.37	11.54	11.71	11.71	11.66	11.49	11.32	11.15	10.99	10.83
5.31	3.39																	
	4.29	5.59	6.92	7.02	7.12	7.23	7.34	7.45	7.56	7.67	7.78	7.75	7.68	7.57	7.46	7.35	7.24	7.13
2.50	1.24																	
	2.13	2.78	3.43	3.48	3.54	3.59	3.64	3.70	3.75	3.81	3.86	3.85	3.81	3.76	3.70	3.65	3.59	3.54
1.24	0.62																	

THE SYSTEM RESPONSE IS

	0.62	1.24	1.37	1.90	1.93	1.95	1.94	2.02	2.05	2.08	2.11	2.12	[illegible]	2.08	2.05	2.02	1.99	1.95	1
1.24	0.62																		
	1.24	4.01	5.31	5.39	5.47	5.55	5.63	5.72	[illegible]	[illegible]	[illegible]	5.02	[illegible]	5.89	5.80	5.72	5.63	5.55	5
1.31	1.24																		
	1.87	5.31	8.78	8.91	9.04	9.17	9.31	9.45	9.59	9.73	9.88	9.96	[illegible]	9.73	9.64	9.50	9.36	9.22	9
6.85	3.39																		
	1.90	5.39	8.91	9.04	9.17	9.31	9.45	9.59	9.73	9.88	10.02	10.11	10.08	9.93	9.78	9.64	9.50	9.35	9
6.95	3.44																		
	1.93	5.47	9.04	9.17	9.31	9.45	9.59	9.73	9.88	10.02	10.17	10.26	10.23	10.08	9.93	9.78	9.64	9.50	9
7.05	3.49																		
	1.96	5.55	9.17	9.31	9.45	9.59	9.73	9.88	10.02	10.17	10.32	10.41	10.38	10.23	10.08	9.93	9.78	9.64	9
7.15	3.54																		
	1.99	5.63	9.31	9.45	9.59	9.73	9.88	10.02	10.17	10.32	10.48	10.56	10.53	10.38	10.23	10.08	9.93	9.78	9
7.25	3.59																		
	2.02	5.72	9.45	9.59	9.73	9.88	10.02	10.17	10.32	10.48	10.63	10.72	10.69	10.53	10.38	10.23	10.08	9.93	9
7.37	3.65																		
	2.05	5.80	9.59	9.73	9.88	10.02	10.17	10.32	10.48	10.63	10.79	10.88	[illegible]	10.69	10.53	10.38	10.23	10.08	9
7.43	3.70																		
	2.08	5.89	9.73	9.88	10.02	10.17	10.32	10.48	10.63	10.79	10.95	11.04	11.01	10.84	10.69	10.53	10.38	10.23	10
7.59	3.76																		
	2.11	5.98	9.88	10.02	10.17	10.32	10.48	10.63	10.79	10.95	11.11	11.20	11.17	11.01	10.84	10.69	10.53	10.38	10
7.71	3.81																		
	2.12	6.02	9.96	10.11	10.26	10.41	10.56	10.72	10.88	11.04	11.20	11.30	11.26	11.10	10.94	10.79	10.62	10.45	10
7.78	3.85																		
	2.11	5.98	9.93	10.08	10.23	10.38	10.53	10.69	10.81	11.01	11.17	11.25	11.23	11.05	10.90	10.74	10.59	10.43	10
7.75	3.86																		
	2.08	5.89	9.78	9.93	10.08	10.23	10.38	10.53	10.69	10.84	11.01	11.17	[illegible]	10.90	10.74	10.59	10.43	10.28	10
7.64	3.81																		
	2.05	5.80	9.64	9.78	9.93	10.08	10.23	10.38	10.53	10.69	10.84	10.98	[illegible]	10.74	10.59	10.43	10.28	10.13	9
7.53	3.75																		
	2.02	5.72	9.50	9.64	9.78	9.93	10.08	10.23	10.38	10.53	10.69	10.79	[illegible]	10.59	10.43	10.28	10.13	9.98	9
7.42	3.70																		
	1.99	5.63	9.36	9.50	9.64	9.78	9.93	10.08	10.23	10.38	10.53	10.62	10.59	10.43	10.28	10.13	9.98	9.84	9
7.31	3.64																		
	1.96	5.55	9.22	9.35	9.50	9.64	9.78	9.93	10.08	10.23	10.38	10.46	10.43	10.28	10.13	9.98	9.84	9.69	9
7.21	3.59																		
	1.93	5.47	9.09	9.22	9.36	9.50	9.64	9.78	9.93	10.08	10.23	10.31	10.28	10.13	9.98	9.84	9.69	9.55	9
7.10	3.54																		
	1.90	5.39	8.95	9.09	9.22	9.35	9.50	9.64	9.78	9.93	10.08	[illegible]	10.13	9.98	9.84	9.69	9.55	9.41	9
7.00	3.48																		
	1.87	5.31	8.82	8.95	9.09	9.22	9.36	9.50	9.64	9.78	9.93	10.01	9.98	9.84	9.69	9.55	9.41	9.27	9
6.89	3.43																		
	1.24	4.01	6.85	6.95	7.05	7.15	7.25	7.37	7.48	7.59	7.71	[illegible]	7.75	7.64	7.53	7.42	7.31	7.21	7
5.57	2.78																		
	0.62	1.24	3.39	3.44	3.49	3.54	3.59	3.65	3.70	3.75	3.81	3.85	3.86	3.81	3.75	3.70	3.64	3.59	3
2.73	2.13																		

THE INPUT PATTERN IS

2.397895 , 0.6931472 , 0.6931472 , 0.6931472 , 0.6931472 , 0.6931472 , 0.6931472 ,
0.6931472

THE SYSTEM RESPONSE IS

2.13	2.78	3.43	3.48	3.54	3.59	3.64	3.70	3.75	3.81	3.86	3.85	3.81	3.76	3.70	3.65	3.59	3.54
1.24	0.62																
2.78	4.06	5.36	5.44	5.52	5.60	5.68	5.77	5.85	5.94	6.03	6.02	5.98	5.89	5.80	5.72	5.63	5.55
2.50	1.24																
3.43	5.36	7.31	7.42	7.53	7.64	7.76	7.87	7.99	8.11	8.23	8.23	8.17	8.05	7.94	7.82	7.71	7.59
3.77	1.87																
3.48	5.44	7.42	7.53	7.64	7.76	7.87	7.99	8.11	8.23	8.35	8.35	8.30	8.17	8.05	7.94	7.82	7.71
3.83	1.90																
3.54	5.52	7.53	7.64	7.76	7.87	7.99	8.11	8.23	8.35	8.47	8.48	8.42	8.30	8.17	8.05	7.94	7.82
3.89	1.93																
3.59	5.60	7.64	7.76	7.87	7.99	8.11	8.23	8.35	8.47	8.60	8.60	8.54	8.42	8.30	8.17	8.05	7.94
3.95	1.96																
3.64	5.68	7.76	7.87	7.99	8.11	8.23	8.35	8.47	8.60	8.73	8.73	8.67	8.54	8.42	8.30	8.17	8.05
4.00	1.99																
3.70	5.77	7.87	7.99	8.11	8.23	8.35	8.47	8.60	8.73	8.85	8.86	8.80	8.67	8.54	8.42	8.30	8.17
4.06	2.02																
3.75	5.85	7.99	8.11	8.23	8.35	8.47	8.60	8.73	8.85	8.99	8.99	8.93	8.80	8.67	8.54	8.42	8.30
4.13	2.05																
3.81	5.94	8.11	8.23	8.35	8.47	8.60	8.73	8.85	8.99	9.12	9.12	9.06	8.93	8.80	8.67	8.54	8.42
4.19	2.08																
3.86	6.03	8.23	8.35	8.47	8.60	8.73	8.85	8.99	9.12	9.25	9.26	9.19	9.06	8.93	8.80	8.67	8.54
4.25	2.11																
3.85	6.02	8.23	8.35	8.48	8.60	8.73	8.86	8.99	9.12	9.26	9.26	9.20	9.07	8.93	8.80	8.68	8.55
4.27	2.12																
3.81	5.98	8.17	8.30	8.42	8.54	8.67	8.80	8.93	9.06	9.19	9.20	9.14	9.00	8.87	8.74	8.62	8.49
4.25	2.11																
3.76	5.89	8.05	8.17	8.30	8.42	8.54	8.67	8.80	8.93	9.06	9.07	9.00	8.87	8.74	8.62	8.49	8.37
4.19	2.08																
3.70	5.80	7.94	8.05	8.17	8.30	8.42	8.54	8.67	8.80	8.93	8.93	8.87	8.74	8.62	8.49	8.37	8.24
4.13	2.05																
3.65	5.72	7.82	7.94	8.05	8.17	8.30	8.42	8.54	8.67	8.80	8.80	8.74	8.62	8.49	8.37	8.24	8.12
4.06	2.02																
3.59	5.63	7.71	7.82	7.94	8.05	8.17	8.30	8.42	8.54	8.67	8.68	8.62	8.49	8.37	8.24	8.12	8.00
4.00	1.99																
3.54	5.55	7.59	7.71	7.82	7.94	8.05	8.17	8.30	8.42	8.54	8.55	8.49	8.37	8.24	8.12	8.00	7.89
3.95	1.96																
3.49	5.47	7.48	7.59	7.71	7.82	7.94	8.05	8.17	8.30	8.42	8.42	8.37	8.24	8.12	8.00	7.89	7.77
3.89	1.93																
3.44	5.39	7.37	7.48	7.59	7.71	7.82	7.94	8.05	8.17	8.30	8.30	8.24	8.12	8.00	7.89	7.77	7.66
3.83	1.90																
3.39	5.31	7.26	7.37	7.48	7.59	7.71	7.82	7.94	8.05	8.17	8.18	8.12	8.00	7.89	7.77	7.66	7.55
3.77	1.87																
1.24	2.50	3.77	3.83	3.89	3.95	4.00	4.06	4.13	4.19	4.25	4.27	4.25	4.19	4.13	4.06	4.00	3.95
2.50	1.24																
0.62	1.24	1.87	1.90	1.93	1.96	1.99	2.02	2.05	2.08	2.11	2.12	2.11	2.08	2.05	2.02	1.99	1.96
1.24	0.62																